COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Written Assignment # 5
Distributed: May 8, 2003
Due: May 19, 2003 at 5:00PM

Note: Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page.

Problem 1. (50 pts) Give a polynomial time reduction from the search version of the Hamiltonian cycle problem to the decision version of the Hamiltonian cycle problem. For your convenience, the Hamiltonian cycle decision and search problems are defined below:

Decision Problem: Given a graph $G = (V, E)$, output *true* if G contains a Hamiltonian cycle; otherwise output *false*.

Search Problem: Given a graph $G = (V, E)$, output the sequence of edges in some Hamiltonian cycle, if G contains a Hamiltonian cycle; otherwise output that G has no Hamiltonian cycle.

In other words, you need to do the following:

(a) Suppose that there exists an algorithm A_1 to solve the Hamiltonian cycle decision problem that runs in time polynomial in the length of the input to A_1. Using algorithm A_1 as a subroutine, design an algorithm A_2 to solve the Hamiltonian cycle search problem that runs in time polynomial in the length of the input to A_2. You should briefly describe the idea behind your algorithm and give the pseudo-code.

Note: For the purposes of this and the other parts of this question you may consider the *length of the input* of graph $G = (V, E)$ to be $|V| + |E|$.

(b) Prove the correctness of algorithm A_2 designed in part (a).

(c) Show that algorithm A_2 runs in time polynomial in the length of the input to A_2.

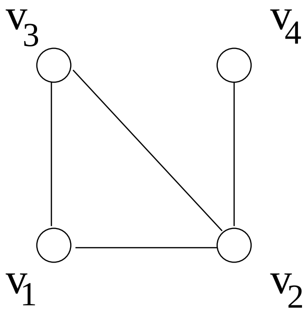
Hint: Given an algorithm for solving the decision problem how can you decide if a particular edge $(u, v) \in E$ is part of some Hamiltonian cycle? This might involve modifying graph G.

Problem 2. (25 pts) Show that the Hamiltonian cycle (HC) problem is NP-complete. You may use the fact that the Hamiltonian path (HP) problem is NP-complete. Here are the problem definitions.

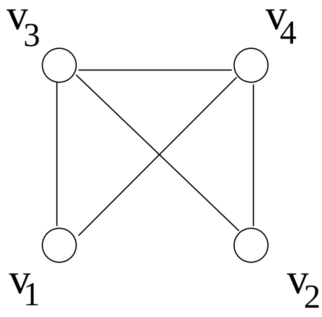
Hamiltonian Cycle (HC): Given an undirected graph $G = (V, E)$, is there a cycle that visits all the vertices of G exactly once.

Hamiltonian Path (HP): Given an undirected graph $G = (V, E)$, is there a simple path that visits all the vertices of G exactly once.

For example, graph G_1 has a Hamiltonian path $v_3 \to v_1 \to v_2 \to v_4$ and graph G_2 has a Hamiltonian cycle $v_1 \to v_3 \to v_2 \to v_4 \to v_1$.



G_1



G_2

Problem 3. (25 pts) (CLRS 34.5-5) The **Set-Partition problem** takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and $\overline{A} = S - A$, such that $\sum_{x \in A} x = \sum_{x \in \overline{A}} x$. Show that the set-partition problem is NP-Complete.

Hint. For the second part of the NP-Completeness proof try showing that

$$\text{Subset-Sum} \leq_P \text{Set-Partition}$$

where Subset-Sum is described at the beginning of section 34.5.5 of CLRS.