COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Revised: March 6, 2003, 6:30PM

1. Given an undirected graph $G = (V, E)$, recall that the complement, $\overline{G}$, is a graph $(V, E')$ such that for all $u \neq v$, $\{u, v\} \in E'$ if and only if $\{u, v\} \notin E$. Prove that either $G$ or $\overline{G}$ is connected.

2. Let $G = (V, E)$ be a connected undirected graph. Let $s$ be any vertex of $V$ and run the BFS algorithm on $G$ starting at $s$. Show that if $\{u, v\}$ is any edge of $E$ then $|d(u) - d(v)| \leq 1$.

3. An (undirected) graph $G = (V, E)$ is bipartite if there exists some $S \subset V$ such that, for every edge $\{u, v\} \in E$, either (i) $u \in S$, $v \in V - S$ or (ii) $v \in S$, $u \in V - S$.

Let $G = (V, E)$ be a connected graph. Design an $O(n + e)$ algorithm that checks whether $G$ is bipartite. \textit{Hint: Run BFS}.

How can you modify your algorithm so that it also works for unconnected graphs?

4. Give an example of a directed graph $G$ in which a vertex $u$ of $G$ ends up in a depth-first tree containing only $u$, even though $u$ has both incoming and outgoing edges. Your example graph should have no self-loops.