1. Prove that $P \subseteq \text{Co-NP}$. 

2. Prove that if $NP \neq \text{Co-NP}$ then $P \neq NP$. 

3. For each of the following assertions, indicate whether it is True: known to be true, False: known to be false, or Unknown: unknown based on our current scientific knowledge. In each case provide a short explanation for your answer.

   (i) No problems in NP can be solved in polynomial time.
   (ii) Every NP-complete problem requires at least exponential time to be solved.
   (iii) $X$ is in NP and $X \leq_P \text{SAT}$. Then $X$ is NP-complete.

4. Given an undirected graph $G = (V, E)$, a feedback vertex set is a subset of vertices such that every simple cycle in $G$ passes through one of these vertices. The feedback vertex set problem (FVS) is: Given a graph $G$ and an integer $k$, does $G$ contain a feedback vertex set of size at most $k$?

   Show that FVS is in NP. That is, given a graph $G$ that has a FVS of size $k$, give a certificate, and show how you would use this certificate to verify the presence of a FVS of size $k$ in polynomial time.

   (Hint: The certificate should be a set $V' \subseteq V$ with $|V'| = k$. You need to show a polynomial time algorithm that tests whether $V'$ is a FVS or not. Note that since a graph can have exponentially many cycles you cannot just do the simple thing of checking every cycle.)

5. The set cover problem is: Given a finite set $X$ and a collection of sets $F$ whose elements are chosen from $X$, and given an integer $k$, does there exist a subset $C \subseteq F$ of $k$ sets such that

   $$X = \bigcup_{S \in C} S.$$ 

   Prove that the set cover problem is NP-complete. (Hint: Reduce from Vertex-Cover.)