GRAPH-BASED OPTIMAL CROSS SECTION BOUNDARY FOR VESSEL SEGMENTATION AND STENOSIS QUANTIFICATION

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ABSTRACT

In this paper, we propose a graph-based method to find the optimal cross section boundary for vessel segmentation. The voxels on the cross sectional plane are assumed to lay on the circles around the centerline point. The voxels on the circles with different radii are then transformed to a graph, by which the objective of finding the optimal boundary is converted to choosing the optimal path in the graph. A new cost function for the edge cost of the graph is proposed to obtain a smooth, optimal boundary of the cross section. Based on the optimal cross section boundary, we also propose a method for stenosis detection and quantification. The proposed method for segmentation and stenosis detection has been evaluated to be accurate and highly computationally efficient.

Index Terms— Vessel Segmentation, Graph-Based Optimal Path, Stenosis Quantification.

1. INTRODUCTION

In recent years, a variety of methods have been proposed for vessel segmentation [1]. Some of them extract the vessel surface directly, for example, 3D active contour methods. Some other methods trace centerline curve in 3D space. They both have their own advantages and drawbacks. As described in [2], surface alone can be insufficient in some applications. For centerline tracing methods, post-processing may be needed for particular application. To solve these problems, methods in [3, 4, 5] incorporate the cross sectional radius as the 4th dimension together with the 3D coordinates of the voxels. Thus when the tracking procedure ends, the radii of the vessel are obtained together with the centerline. With the radii related to the centerline points, approximate surface of the vessel is also obtained.

However, the assumption of circular cross section for vessels is just an approximation. This will sometimes lead to inaccurate segmentation for the irregular or elliptical cross sections. In most problematic vessel regions, such as aneurysm or stenosis, the cross section is usually not regular. The circular assumption then will cause the ignorance of the real degree of stenosis or aneurysm. As shown in Fig.1, the obtained circular cross section sometimes may not represent the real cross section very well. As a result, the reconstructed vessel image may not represent the original shape properly, which may lead to inaccurate diagnosis for the patients with aneurysm or stenosis. For solving this problem, we propose a new method
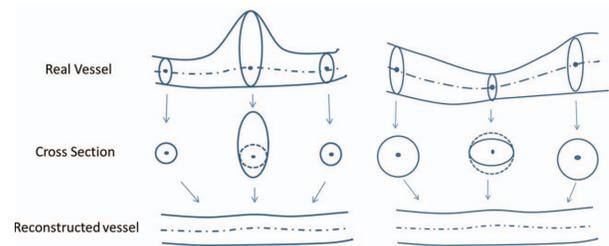

Fig. 1. Problem of Circular Cross Section Assumption.

which does not make any assumption of the shape of the cross section. With this method, the surface reconstructed will be more accurate by trying to keep the original shape of cross section. The reconstructed result obtained by the new method, as a result, may be more reliable for diagnosis and treatment planning.

Vessel stenosis is a kind of abnormal narrowing which may lead to inadequate blood supply or even sudden death. In recent years, many methods have been proposed for stenosis detection and quantification [6, 7, 8, 9]. There are two major types of quantification methods, minimal radius based methods and area based methods. In most cases, the shape of the cross sections in stenosis region is complicated, making minimal radius or area based methods not good for quantification. In this paper, based on the accurate cross section segmentation, we propose a method for stenosis detection and quantification.

2. ACCURATE VESSEL SEGMENTATION

In this section, we propose a new vessel segmentation method which does not make any assumption about the shape of the cross section. The surface reconstructed will be more accurate and more reliable for diagnosis and treatment planning. With

the proposed method, both the centerline points and associated accurate cross section boundaries are obtained simultaneously. Minimum average-cost method presented in [10] is used to select the centerline points. For each centerline point, a reference radius is also obtained. With the centerline point, we then focus on finding the optimal cross section boundary, so as to obtain the accurate surface for vessels.

2.1. Graph-Based Optimal Cross Section

Given a centerline point c and its previous point pre, we assume the plane of the cross section associated with c is perpendicular with the direction of $c-pre$. As shown in Fig.2(a), we assign each voxel in ROI on the plane to the nearest circle. Then the voxels on the circles are transformed to nodes in an directed graph as shown in Fig.2(b). For each row of the graph, the first node and the last node are the same voxel in the original image. For each node $p_{ab}(0 \leq a \leq K, 0 \leq b \leq J)$ in the graph, possible successor node p_{xy} satisfies $1 \geq x - a \geq -1$ and $1 \geq y - b \geq 0$. So the path will not go back to the left. With this transformation, we convert the objective for finding the boundary of the cross section to choosing the optimal path p_{i0} to p'_{i0} in the directed graph.

There are many algorithms that can be used to find the optimal path, including dynamic programming, Bellman-Ford, Floyd and single source shortest path methods. In this application, since the ROI is not very large and the objective is just choosing the optimal path i with the smallest cost from p_{i0} to $p'_{i0}(0 \leq i \leq K)$, we choose the Dijkstra algorithm to find the shortest path for all p_{i0} to p'_{i0} and then choose the one with minimal cost as the optimal path. It can be easily proved that the solution is optimal based on the given graph.
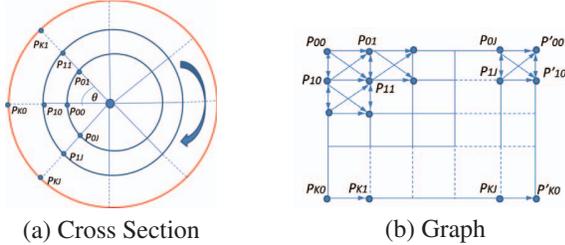

(a) Cross Section   (b) Graph

Fig. 2. Transformation of Cross Section to Graph

2.2. Cost Function

As we have stated in Section 2.1, the path obtained with the proposed method is optimal. The segmentation result, however, also depends on the cost function of the graph. In other words, the optimal path is based on the cost function. With poor cost function, the segmentation result may not be good although the path is optimal for that directed graph with particular edge cost. Here we propose a new cost function for the edges in the graph.

For the cost of edge from p_{ab} and p_{xy}, the cost function is defined as follows:

$$Cost(p_{ab}, p_{xy}) = e^{-(G_{ab}+G_{xy})/\omega_1 - F(p_{ab},p_{xy})/\omega_2}. \quad (1)$$

The exponential function here is used to make the cost of edge nonnegative. We use ω_1 and ω_2 to enlarge the difference of the cost and balance the two parts as weights. The use of G_{ab} and G_{xy} is inspired by the method in [11].

$$G_{ab} = \boldsymbol{g}_{ab} \cdot \boldsymbol{u}_{ab}, \;\; \boldsymbol{u}_{ab} = unit(p_{ab} - c), \;\; unit(\boldsymbol{m}) = \boldsymbol{m}/|m|,$$

$\boldsymbol{g}_{ab}$ here denotes the gradient vector of p_{ab}, c is the centerline point, and operator $\cdot$ is dot product. Different from [11], we do not use the sum of G_{ab} on the circle to measure the the medialness. Instead we use G_{ab} directly as a measure of whether p_{ab} is a good candidate on the surface, and same for G_{xy}. $F(p_{ab}, p_{xy})$ in Equation 1 is defined as follows:

$$v = unit(c-pre) \times unit(p_{ab}-p_{xy}), F(p_{ab}, p_{xy}) = \boldsymbol{v} \cdot (\boldsymbol{g}_{ab} + \boldsymbol{g}_{xy}).$$

Operator $\times$ here denotes cross product. $F(p_{ab}, p_{xy})$ is the response for p_{ab} and p_{xy} being neighboring nodes on the boundary. As shown in Fig. 3, $\boldsymbol{v}$ here denotes the direction perpendicular with $p_{ab} - p_{xy}$. The response of $F(p_{ab}, p_{xy})$ should be high if p_{ab} and p_{xy} are neighboring nodes on the boundary, no matter whether they are on the same circle or not.
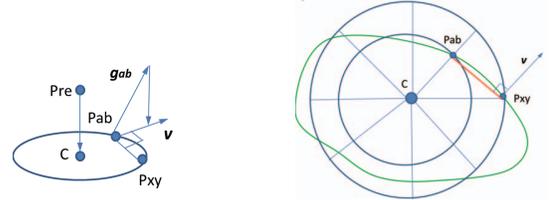

Fig. 3. Illustration of the Cost Fuction

3. STENOSIS DETECTION AND QUANTIFICATION

Current methods for stenosis quantification are either based on minimal radius or area of cross section. However, different shapes with same minimal radius or same area may represent different stenosis degrees. Fig.4(a) shows a normal reference shape of cross section. In Fig.4(b), the two round shape cross sections have the same minimal radius, but obviously, the one with smaller area is more seriously narrowing in all directions, compared with the shape in Fig.4(a). Area based quantification methods also have problems in some cases. The radii of healthy vessels sometimes may also decrease and then increase while tracing towards the distal part. As shown in Fig.4(c), the shape on the right, in some cases, may not be a real stenosis area but just normal narrowing. The left shape, however, is a real stenosis cross section with the horizontal direction narrowed. The area based methods cannot distinguish the difference between the two shapes. In this section, we pro-
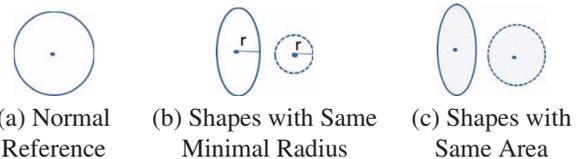

(a) Normal   (b) Shapes with Same   (c) Shapes with
Reference   Minimal Radius   Same Area

Fig. 4. Cross Sections with Same Minimal Radius or Area

pose a stenosis quantification method based on the optimal cross section boundary obtained in Section 2. The accurate cross section together with a reference radius associated with each centerline point is obtained. First, the reference radii are used to detect the candidate stenosis areas. The local minima and maxima are found alternately, and then a threshold is used to remove the candidate with small radius changes. Then, the accurate cross section boundary for candidate local minimum point is used to perform quantification. For a stenosis candidate centerline point c, R is the average radius of the cross sections of two neighboring local maxima, R_{left} and R_{right}. Then the new quantification is defined as follows:

$$R_j = R - |c - p_{Ij}|, SG(c) = \frac{1}{N \times R^2} \sum_j [R_j^2 + (R_j - \bar{R})^2]$$

p_{Ij} here denotes the jth point on the optimal path of the cross section, which is associated with the candidate centerline point with its reference radius as local minimum. $\bar{R}$ represents the mean of R_j. With this quantification method, SG, which denotes the stenosis grade, may distinguish the grade of narrowing as well as the variance of the shape.

4. EXPERIMENTS

To evaluate the proposed method, we conduct experiments on synthetic images and a series of clinical CTA images. In the first group of experiments, a synthetic image without noise is generated by setting the intensity of the voxels in the area within the surface to 1200 and that of the voxels outside to 800. Besides, the cross sections of the synthetic images are ellipses. Gaussian noise is added to the this synthetic image to generate a series of noisy synthetic images.
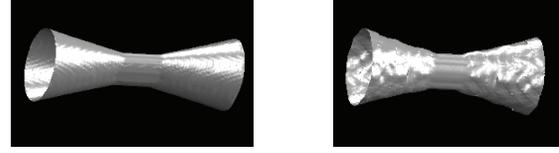
We use accuracy and RMSE (root-mean-square error) as evaluation measures. Suppose voxel $p_k(1 \leq k \leq J \times N)$ is a boundary node obtained by our method. J is the number of nodes on each cross section, and N is the number of cross sections. we label p_k as an accurate voxel when its distance from the real boundary is within 1 voxel spacing, and an inaccurate voxel otherwise. Then accuracy is defined as the percentage of accurate voxels among all the $J \times N$ boundary nodes obtained by our method. We define RMSE as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{k=1}^{J \times N} (\sqrt{C_k} - 1)^2}{J \times N}},$$

where $C_k = (X_k - m)^2/a^2 + (Y_k - n)^2/b^2$ and (X_k, Y_k) are the 2D coordinates of p_k on the cross sectional plane. For a particular cross section, we assume the node (x, y) on the ground truth ellipse in 2D satisfies $(x - m)^2/a^2 + (y - n)^2/b^2 = 1$, where (m, n) is the ellipse center. As shown in Table 1, accuracy and RMSE results are good, showing that our results are very close to the ground truth. Moreover, the rendering surface of the ground truth and that of our estimation are given in Fig. 5. The two surfaces are similar

Table 1. Accuracy on Synthetic Images (σ is noise level)

	WithoutNoise	$\sigma = 200$	$\sigma = 400$	$\sigma = 600$	$\sigma = 800$
Accuracy	100%	100%	99.8504%	98.6595%	98.5233%
RMSE	0.0468	0.0522	0.0578	0.0643	0.0674

(a) Groundtruth Surface   (b) Estimated Surface

Fig. 5. Segmentation on Synthetic Image

although the input synthetic image of the estimated result is noisy ($\sigma = 600$).

The second group of experiments are conducted on several CTA images for coronary artery segmentation and stenosis detection. For finding the optimal cross section boundaries, radii range from $r - 0.9$ to $r + 0.9$ with fixed step of 0.3, and r is the reference radius associated with the centerline point. On each circle, 30 nodes are recorded. As a result, the size of the graph matrix is 7×31. Fig. 6 shows the segmentation results of a coronary artery tree. Fig. 7 gives some
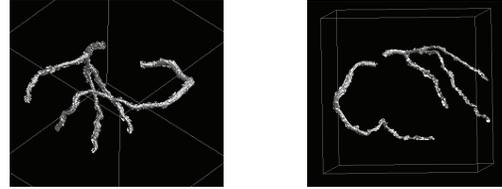

Fig. 6. Segmentation Results for Real Images

experimental results of the optimal path method. In each row, there are 3 images representing the original cross section area, reference circular boundary from the Rotterdam Framework [12], and the optimal boundary obtained with the proposed method, respectively. As shown in the images, the circular boundary, as an approximation, sometimes cannot represent the real boundary very well. But the optimal paths obtained with the proposed method fit the boundaries better in these cases.

For stenosis detection, the local minima and maxima are first found based on the reference radii, and then for each local minimum satisfies $r/R_{left} < T$ and $r/R_{right} < T$ with $T = 0.95$. Quantification is then done to evaluate the narrowing of the cross sections. Fig.8(a) shows an image of the stenosis area, and Fig.8(b) presents the corresponding optimal cross section boundary with SG=0.23 for the local minimum candidate in this stenosis area. We note that SG represents
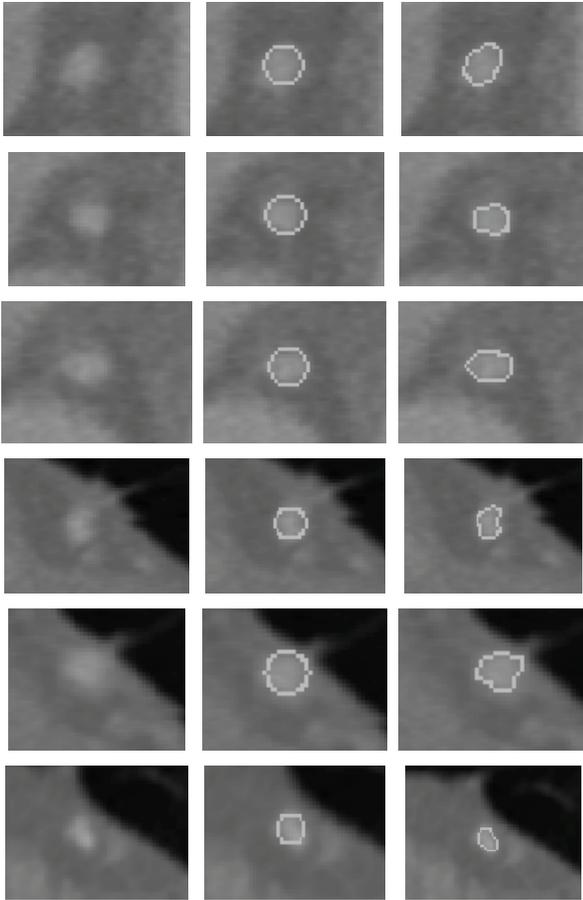
Fig. 7. Segmentation of Cross Sections

method for vessel segmentation. The voxels on the cross sectional plane are transformed to nodes on a graph, by which the objective of finding the optimal boundary is converted to choosing the optimal graph path. A new cost function is also proposed for finding a smooth, optimal boundary in the graph. Based on the optimal cross section boundary, we propose a stenosis detection and quantification method to measure the narrowing of vessel. The proposed segmentation method is highly computationally efficient and achieves promising performance.

6. ACKNOWLEDGEMENT

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the narrowing degree of vessel, and the average SG value for normal cross sections is around 0.03.

The proposed method is highly computationally efficient. The whole segmentation procedure for a coronary artery tree with 4 major branches can be done within 30 seconds on a laptop with a 2.6GHz processor and 4GB RAM. In this procedure, the centerlines, reference radii, and the optimal cross section boundaries are obtained. The automatic stenosis detection and quantification can be done within only 1 second.

(a) Stenosis Area   (b) Cross Sections of Stenosis

Fig. 8. Stenosis Area and Cross Section

5. CONCLUSION

In this paper, we propose a graph-based method to find the optimal cross section boundary, which leads to an accurate