A Decentralized Approach to Sensory Data Integration

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Abstract

In this paper, a decentralized approach based on the team consensus approach and Markovian model is proposed to integrate multisensory data. A team of sensors can estimate the local and global uncentainties utilizing self-entropy and conditional-entropy measures of the sensors. Consensus can be reached based on the initial expected values and "uncertainty" weights assigned by the sensors. The proposed approach is compared with the Baysesian approach via experiments on two independent sensors. Results showed that consensus reached are comparable. However, there are factors that indicated the decentralized approach requires less communication and computational effort to reach consensus among sensors.

1 Introduction

In multisensor based systems, one of the main issues of concern is the reliability of the information received from the various sensors in the system. Researchers have proposed several mathematical models for representing and combining uncertain information [1, 2, 3]. As pointed out in [4], in a team of sensors, two types of uncertainties seem to emerge. The first type of uncertainty is local to the sensor. By observing the state of nature, each sensor gathers information which is then used to estimate the local uncertainty of the sensor, compute the utilities of the set of actions under its disposal, and compute a preference order on the action set. The second type of uncertainty is global, and it manifests in the cooperation of sensors in terms of information exchange. Because the team is assumed to be cooperative, sensors are allowed to intercommunicate the results of their observations. Given the results of its own observations as well as the results of the observations of other sensors, each sensor will be able to reduce its uncertainty level. The latter mechanism mimics how sensors improve their state of information by learning the results of the observations of one another. It has been found for example that depth, motion, shape and colour perception are interrelated [5]. In multisensor systems, this type of dependence may occur, and we refer them as cooperating in a complementary mode. The dependency between observations made by sensors and the state of nature can be considered as a positive type. The stronger this dependency is, the more informative are the observations made by the sensors about the state of nature. This explains why Bayesian estimation procedures would weigh more on observations, which are strongly dependent on the parameter to be estimated, than those which are less dependent.

In this paper, we first review the probabilistic approach to modelling uncertainty and cooperation in multisensor systems [4, 6]. A decentralized approach based on Markovian model is proposed to integrate multisensory data. This approach allows each sensor, treated as an expert, to compute its information locally first. Then, a team consensus will be reached after taking into consideration the dependence between the team members. The proposed approach is demonstrated by a team of two sensors, namely a sonar sensor and a b/w CCD camera.

2 Self Entropy and Conditional Entropy

Shannon's entropy function [7] has been used extensively as a measure of uncertainty. We propose two types of entropy measures, namely, self-entropy and conditional-entropy, to estimate the local and global uncertainties. The self-entropy measures how uncertain the sensor is about its own information or how random the data collected are. The self-entropy of sensor, S_i, denoted by $h_{i|i}(\hat{\theta}_i)$ is given by

$$h_{i|i}(\hat{\theta}_i) = -\sum_{\theta_i \in \Theta} p(\theta_i|\hat{\theta}_i) \log p(\theta_i|\hat{\theta}_i) \tag{1}$$

where $p(\theta_i|\hat{\theta}_i)$ is the probability of occurrence of the state θ_i given the observed state $\hat{\theta}_i$.

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The conditional-entropy, however, is a measure of the state of uncertainty of a sensor given the information of another sensor. This entropy can be used to capture the essence of information relevance exchanged among the team of sensors. For example, if the information measured by one sensor is irrelevant to another sensor, we will find that the conditional-entropy of this sensor equal to its self-entropy. Meaning that information provided by the first sensor does not help the second one to improve its state of uncertainty. The conditional-entropy manifests profoundly the interdependence between sensors. As we will see later, the information provided by one sensor can only decrease the amount of uncertainty of another sensor. The conditional-entropy between sensor, S_i and S_j, denoted by $h_{i|j}(\hat{\theta}_i)$, is given by

$$h_{i|j}(\hat{\theta}_i) = -\sum_{\hat{\theta}_i \in \Theta} p(\hat{\theta}_j|\hat{\theta}_i) \sum_{\theta \in \Theta} p(\theta|\hat{\theta}_i, \hat{\theta}_j) \log p(\theta|\hat{\theta}_i, \hat{\theta}_j)$$

where $p(\theta|\hat{\theta}_i, \hat{\theta}_j)$ is the joint and conditional probability of occurrence of the state θ given the observed state $\hat{\theta}_i$ and $\hat{\theta}_j$. $h_{i|j}$ is not necessarily equal to $h_{j|i}$. Note that for any two sensors, S_i and S_j, the following conditions hold:

- $h_{i|i} \geq h_{i|j}$;
- if they are independent, then $h_{i|i} = h_{i|j}$

As an example, consider the case of two sensors S_i and S_j, cooperating to measure the value of state of nature θ. Sensor S_j computes its self uncertainty, i.e., self-entropy: $h_{j|j}$ and the conditional uncertainty of sensor S_j assuming that it has received some information from sensor S_i, i.e., the conditional-entropy: $h_{j|i}$. If the information of sensor S_i happens to be relevant for sensor S_j, then we expect that $h_{j|i}$ to be less than $h_{j|j}$, which means that this information contribute to reducing the uncertainty of sensor S_j. Otherwise, sensor S_j should maintain its uncertainty level, namely $h_{j|j}$. Same argument applies to sensor S_i. These entropy measures will contribute towards the determination of a "weight" assignment which will reflect the self and joint confidence of the sensors in measuring the value of the state of nature θ. These measures are updated whenever new information relevant to the sensory task is gathered. Figure 1 depicts these measures diagrammatically.

In general, for a team of N sensors, these measures can be expressed in matrix form:

$$H = \begin{bmatrix} h_{1|1} & h_{2|1} & \cdots & h_{N|1} \\ h_{1|2} & h_{2|2} & \cdots & h_{N|2} \\ \cdots & \cdots & \cdots & \cdots \\ h_{1|N} & h_{2|N} & \cdots & h_{N|N} \end{bmatrix}.$$

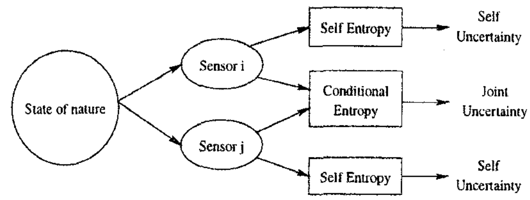


Figure 1: Modelling Sensor Interdependence

Note that the column vector gives the self- and conditional-entropy measures of sensor S_i. Thus if all sensors are independent, H is reduced to a row vector.

Generally speaking, the computation of these measures is application dependent. And to a certain extent requires some degree of intelligence in the sense that sensors should be able to learn about each other from "experience". For example, in some applications the conditional entropy measures (or conditional probabilities) are specified in terms of their pair wise correlation coefficients. In other applications, nevertheless, it would be necessary to have the estimates of these probabilities evolve with time. In this case, one possibility is to have the team start with noninformative conditional probabilities as an initial estimate, and as the team of sensors work together they will be able to converge to more informative estimates.

3 Team Consensus Approach

In this section, we adapt the team consensus approach proposed in [3] and explain how the information variation based uncertainty representation scheme can be used to facilitate the consensus team. In this model, each sensor must first assess its own initial expected utility function, $U_i^{(0)}(\gamma), \ \forall \gamma \in \Gamma$, based on the information it gathered about the value of the state of nature θ. It is then confronted with the utilities of the other team members and revises its own utility in light of the others by making an assessment of each team member's relative importance, expertise, etc. Specifically, at the k^{th} revision, each revised expected utility is deemed to be of the form

$$U_i^{(k)}(\gamma) = \sum_{j=1}^{N} W_{i,j}(\gamma) U_j^{(k-1)}(\gamma), \quad W_{i,j}(\gamma) \geq 0 \qquad (2)$$

where $W_{i,j}(\gamma)$ is a weight assigned by sensor, S_i to sensor, S_j, and $\sum_{j=1}^{N} W_{i,j}(\gamma) = 1, \ \forall i,j = 1,\ldots,N$. It should be noted that W is a stochastic matrix because each element $W_{i,j}(\gamma)$ is a nonnegative and the sum of elements in any given row is 1. Furthermore, if we let $U^{(k-1)}(\gamma)$ and $U^{(k)}(\gamma)$ denote the column vectors given as

$$U^{(k-1)}(\gamma) = (U_1^{(k-1)}, U_2^{(k-1)}, \ldots, U_N^{(k-1)})$$
$$U^k(\gamma) = (U_1^{(k)}, U_2^{(k)}, \ldots, U_N^{(k)})$$

It follows from Equation (2) that

$$U^{(k)}(\gamma) = W(\gamma)U^{(k-1)}(\gamma) = W^k(\gamma)U^{(0)}(\gamma)$$

where $U^{(0)}(\gamma)$ is a vector of the initial utilities of the team members, i.e., $U^{(0)}(\gamma) = (U_1^{(0)}, \ldots, U_N^{(0)})$. The initial utility of each sensor is assessed based on its local observations.

Given that each sensor revises its opinion in this manner, to be consistent each should then update its own utility in light of the revisions made by the others and the process continued until further revision no longer changes the expected utility of any member. The revised expected utilities of the N sensors will converge to each other if and only if there is a utility $u^*(\gamma)$ such that

$$\lim_{k\to\infty} U_i^{(k)}(\gamma) = u^*(\gamma) \tag{3}$$

$\forall i = 1, \ldots, N$.

In other words, the team will reach a consensus if and only if all N elements of $U^{(k)}(\gamma)$ converge to the same limit as k approaches infinity. If we denote by $W_{i,j}^{(k)}(\gamma)$ the elements in row i and column j of the matrix $W^k(\gamma)$, it follows from Equation (3) that a consensus is reached if and only if there exists a vector $\mathcal{K}(\gamma) = (\kappa_1, \ldots, \kappa_N)$ such that, for $\forall i = 1, \ldots, N$ and $\forall j = 1, \ldots, N$,

$$\lim_{k\to\infty} W_{i,j}^{(k)}(\gamma) = \kappa_j(\gamma) \tag{4}$$

If Equation (4) is satisfied for every value i and j, then $\kappa_1, \ldots, \kappa_N$ are necessarily nonnegative and $\sum_{i=1}^N \kappa_i = 1$. Therefore, when a consensus is reached its value will be

$$u^*(\gamma) = \sum_{i=1}^{N} \kappa_i(\gamma) U_i^{(0)}(\gamma) \tag{5}$$

We state the condition for reaching consensus without proof [8, 9]: *if there exists a positive integer k such that every element in at least one column of the matrix $W^k(\gamma)$ is positive, then a consensus is reachable.* The following theorem [3] states how the utility value that forms the team consensus can be explicitly calculated.

Theorem 1 Suppose that a consensus is reached and let $\sum_{i=1}^N \kappa_i U_i^{(0)}(\gamma)$ denote the value of this consensus, then $\mathcal{K}(\gamma) = (\kappa_1, \ldots, \kappa_N)$ is the unique stationary probability vector.

Thus, the value of the vector $\mathcal{K}(\gamma)$ used to calculate the team consensus is determined by solving the linear equations

$$\mathcal{K}(\gamma)W(\gamma) = \mathcal{K}(\gamma) \tag{6}$$

Subject to

$$\sum_{i=1}^{N} \kappa_i(\gamma) = 1.$$

Given the entropy matrix H, how can each sensor of the team determine appropriate weights for itself and other sensors? One objective could be set such that the sum of the squares of all entropies in each row in the entropy matrix is minimized. That is, for each sensor, to minimize the sum of squares of its self-entropy and the conditional-entropy associated with other sensors. This implies that each sensor will assign high weights to sensors with low conditional-entropy and low weights to those with high conditional-entropies. The minimization problem may be stated as follows:

Minimize

$$R_i(\gamma) = \sum_{s_j \in S} W_{i,j}^2(\gamma) \times h_{j|i}^2(\gamma)$$

Subject to

$$\sum_{j=1}^{N} W_{i,j}(\gamma) = 1,$$

$$W_{i,j}(\gamma) > 0.$$

Optimizing the above objective function will yield the following optimal weighting scheme:

$$W_{i,j}(\gamma) = \frac{1}{h_{j|i}^2(\gamma) \sum_{k \in S} \frac{1}{h_{k|i}^2(\gamma)}}. \tag{7}$$

From Equation (7), it can seen that the value under the summation sign in the denominator is the same for all $W_{i,j}(\gamma)$. Therefore $W_{i,j}(\gamma)$ varies inversely with $h_{j|i}^2(\gamma)$.

4 Markovian Model of a Team of Two Sensors

In this section, we shall consider a team of two sensors, S_1 and S_2. The decision process is modelled by the Markovian model (Figure 2).

From the team consensus model, the expected utility at the k^{th} iteration is given by

$$U^k(\gamma) = W(\gamma)U^{(k-1)}(\gamma)$$

$$= \begin{bmatrix} W_{11}(\gamma) & W_{12}(\gamma) \\ W_{21}(\gamma) & W_{22}(\gamma) \end{bmatrix} \begin{bmatrix} U_1^{(k-1)}(\gamma) \\ U_2^{(k-1)}(\gamma) \end{bmatrix}$$

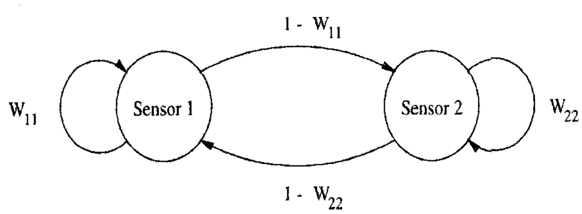


Figure 2: Makovian Model for Two Sensors

where $W_{12}(\gamma) = 1 - W_{11}(\gamma)$ and $W_{21}(\gamma) = 1 - W_{22}(\gamma)$. When consensus is reached, we have, from Equation (5)

$$u^*(\gamma) = \kappa_1(\gamma)U_1^{(0)}(\gamma) + \kappa_2(\gamma)U_2^{(0)}(\gamma) \tag{8}$$

and

$$\begin{bmatrix} \kappa_1(\gamma) & \kappa_2(\gamma) \end{bmatrix} W(\gamma) = \begin{bmatrix} \kappa_1(\gamma) & \kappa_2(\gamma) \end{bmatrix} \tag{9}$$

where $\kappa_1(\gamma) + \kappa_2(\gamma) = 1$. Solving Equation (9) gives,

$$\begin{aligned} \kappa_1(\gamma) &= \frac{1 - W_{22}(\gamma)}{2 - W_{11}(\gamma) - W_{22}(\gamma)} \\ \kappa_2(\gamma) &= \frac{1 - W_{11}(\gamma)}{2 - W_{11}(\gamma) - W_{22}(\gamma)} \end{aligned} \tag{10}$$

and relating W to the entropy matrix H (Equation 7) and assuming the two sensors are independent, we have

$$\begin{aligned} W_{11}(\gamma) &= \frac{1}{h_{1|1}^2(\gamma)(\frac{1}{h_{1|1}^2(\gamma)} + \frac{1}{h_{2|2}^2(\gamma)})}, \\ W_{22}(\gamma) &= \frac{1}{h_{2|2}^2(\gamma)(\frac{1}{h_{1|1}^2(\gamma)} + \frac{1}{h_{2|2}^2(\gamma)})} \end{aligned} \tag{11}$$

where $0 \leq h_{1|1}(\gamma) \leq 1$ and $0 \leq h_{2|2}(\gamma) \leq 1$. Note that weights for each sensor is inversely proportional to the square of the self-entropy of each sensor. The larger the self-entropy, the smaller the weight.

5 Experimental Results

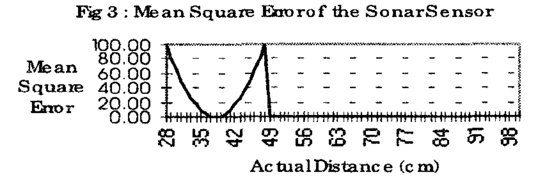
This section demonstrates the Markovian model by considering a team of one sonar sensor and a b/w CCD camera. The aim of integration is to complement the weaknesses of sonar sensors and CCD cameras when they are estimating the object distance alone. In the experiment, the sonar sensor and the CCD camera are mounted on the gripper of a robot arm. Both sensors contribute to the decision process and finally reach a consensus, which is the estimated distance between the gripper and the object. The team consensus value, $u^*(\gamma)$, can then be fed into the robot controller for the next step of action.

In our experiment, a commercial sonar sensor is used. It has a limitation on the range of measurement from 0.49m to 12m, within 1 % of error. For objects closer than 0.49m, it gives readings with large error. Thus, we use a b/w CCD camera which is usually for object recognition rather than distance measure, to compensate the inadequacy of the sonar sensor. By considering the size of a small black circle placed on the object, the CCD camera can estimate the distance of the object observed. This is achieved by measuring the length of the diameter of the circle observed, i.e. the number of pixels along the diameter in the image. The closer the object, the larger the number of pixels in the image. The change in the size of the circle is significantly large as the gripper moves closer to the object. Whereas, the change in the size of the circle is small as the camera moves farther away from the object resulting in larger error of estimation of the distance between object and gripper.

Therefore, this team of two sensors compensate each other in the sense that for measuring distance less than 0.49m, the CCD camera can be expected to give better estimates and vice versa. Assuming these two sensors are independent, we shall estimate the team consensus value, $u^*(\gamma)$, by the decentralized approach and finally compare with the Bayesian approach.

5.1 Sonar Sensor Data

Fig 3 : Mean Square Erorof the SonarSensor



Based on the sonar sensor alone, the initial expected distance, $U_1^{(0)}(\hat{d}_i)$, is given by

$$U_1^{(0)}(\hat{d}_i) = \sum_{d_i \in D} d_i \times P(d_i|\hat{d}_i) \tag{12}$$

where d_i and $\hat{d}_i$ are the true and observed distance respectively. Distances between 28cm to 100cm were "observed" by the sonar sensor. Figure 3 gives the mean square error of the distance observed. It should be noted that for distances ranging between 49cm to 100cm, very good estimates are observed. Whereas error increases when the distance falls below 49cm, the limitation of the sonar sensor given by the manufacturer of the sensor.

5.2 CCD Camera Data

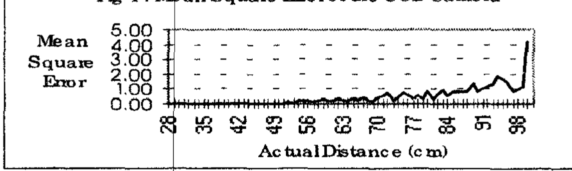


Fig 4 : Mean Square Error of the CCD Camera

Based on the b/w CCD camera alone, the initial expected distance, $U_2^{(0)}(C)$, is given by

$$U_2^{(0)}(C) = \sum_{d_i \in D} d_i \times P(d_i|C) \tag{13}$$

where d_i is the true distance and C is the number of pixels observed by the camera. Again actual distances between 28cm to 100cm were observed by the camera. Similar to the sonar sensor. Figure 4 gives the mean square error of the distance observed. As expected, this sensor gives better estimates for distance below 53cm.

5.3 Decentralized Approach to reach consensus

With the assumption that these two sensors are independent, $W(\gamma)$ and $\mathcal{K}(\gamma)$ can be computed from Equation (10) and (11). From the Equation (1), we have

$$\begin{aligned} h_{1|1}(\hat{d}_i) &= - \sum_{d_i \in D} P(d_i|\hat{d}_i) \log P(d_i|\hat{d}_i) \\ h_{2|2}(C) &= - \sum_{d_i \in D} P(d_i|C) \log P(d_i|C) \end{aligned} \tag{14}$$

Figure 5a and 5b shows the variation of $h_{1|1}(\hat{d}_i)$ and $h_{2|2}(C)$ when the distance ranges from 28cm to 100cm.

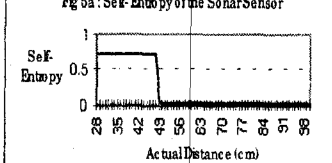


Fig 5a : Self-Entropy of the Sonar Sensor

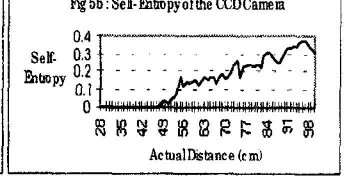


Fig 5b : Self-Entropy of the CCD Camera

Weights ($W_{11}(\hat{d}_i, C)$ and $W_{22}(\hat{d}_i, C)$) of both sensors, κ_1 and κ_2 can be computed based on Equation (11, 10). Hence, consensus can be reached at $u^*(\hat{d}_i, C) = \kappa_1(\hat{d}_i, C)U_1^{(0)}(\hat{d}_i) + \kappa_2(\hat{d}_i, C)U_2^{(0)}(C)$.

Figure 6 shows the fact that the mean square error of the decentralized approach is only between 0 cm and 0.03 cm which is much less than that provided by any single sensor. It demonstrates that the decentralized approach can improve the measurement accuracy, as compared with the performance of each individual sensor.

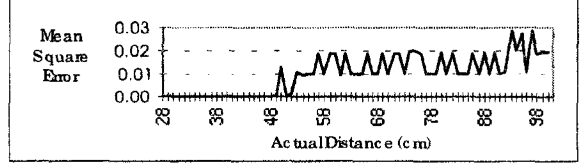


Fig 6 : Mean Square Error of the Decentralized Approach

5.4 Bayesian Approach to reach consensus

The Bayesian model, figure 7, is used as a benchmark to evaluate the performance of the decentralized approach.

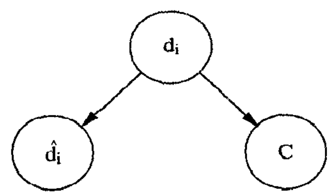


Figure 7 : Bayesian Model of Sonar Sensor and CCD Camera

By Bayes' formulae, the expected distance, given $\hat{d}_i$ and C, is,

$$E[d_i|\hat{d}_i, C] = \sum_{d_i \in D} d_i \times P(d_i|\hat{d}_i, C)$$

$$= \sum_{d_i \in D} d_i \times \frac{P(d_i)P(\hat{d}_i|d_i)P(C|d_i)}{\sum_{d_i \in D} P(d_i)P(\hat{d}_i|d_i)P(C|d_i)} \tag{15}$$

It is valuable to point out that, from Figure 8, the mean square error of the Bayesian approach also ranges from 0 cm to 0.03 cm which is very close to the results given by the decentralized approach.

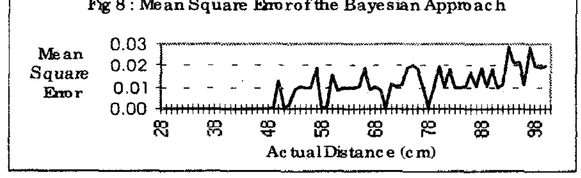


Fig 8 : Mean Square Error of the Bayesian Approach

5.5 Summary

In summary, the decentralized approach, when compared with the Bayesian approach, gives satisfactory mean square error bound in the sonar data and visual data integration.

For a team of two independent or dependent sensors, a consensus value will be $u^*(\hat{d}_i, C) = \kappa_1(\hat{d}_i, C)U_1^{(0)}(\hat{d}_i) + \kappa_2(\hat{d}_i, C)U_2^{(0)}(C)$. From this, it is clearly shown that the decision process can be simplified from an iterative decision process into a "constant time" decision process which involves the initial expected values and weights of each sensor. Equation (5) shows that it is also true for a team of n sensors.

In general, for a team of independent sensors, the initial expected values and weights are calculated based on the local information of each sensor. In other words, each expert in the team takes over the responsibility of estimating the initial expected value and its self-entropy. Both initial expected value and self-entropy will then be sent to a "decision agent" to reach a consensus by linearly combining the initial expected values, Equation (8).

Therefore, the decentralized approach requires less communication and computational effort to reach consensus than the Bayesian approach for the following reasons:

- from Equation (8), only the initial expected values and weights are involved in the data transmission process, and
- from Equation (12, 13, 14), the conditions of probabilities, $\hat{d}_i$ and C, are constant for all d_i in the process of calculating the local entropy and the initial expected values. However, in the Bayesian approach, the conditions of probabilities, d_i, vary with $\hat{d}_i$ and C when computing the expected value in Equation (15).

6 Conclusion and Future Research

In this paper, we have proposed and demonstrated a decentralized approach to integrate multi-sensory data for a team of two independent sensors, namely, a sonar sensor and a b/w CCD camera.

Moreover, the decentralized approach is best suited for the environment that has all the sensors widely separated physically and has noisy channels because the amount of data required to transmission through the channels is much less than that required by the Bayesian approach.

In addition to that, the decentralized approach alleviates the burden of the decision agent. In Bayesian approach, one of the major tasks of decision agent is to maintain and update the conditional probability tables (CTP) of each sensor. These CTPs are solely used by the decision agent to compute the estimated value based on Equation (15). However, in the decentralized approach, the self-entropy and initial expected values are computed by each sensor expert instead of the decision agent. It is obvious that each sensor expert can take over the responsibility of maintaining and updating its own CTP. As a result, the decision agent will have a greater capacity to deal with other robotic tasks.

The approach proposed can be extended to a team of n sensors in general. The difficulty we foresee in the extension lies in "solving" of Equation (6) to compute the values of each κ_i. We shall report the findings in due course.

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