Optimizing Router Placement for Wireless Mesh Deployment

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Abstract—The performance of multi-channel (MRMC) wireless mesh networks (WMNs) is largely affected by interference, which depends on router placement, traffic routing and channel assignment. This work seeks to jointly optimize the placement, routing and channel assignment for mesh deployment given flow requirements. The problem is complex because router placement changes network topology which in turns impacts routing and channel decisions, while routing and channel assignment algorithms affect router placement decision.

We present a novel scheme PRACA (Placement, Routing And Channel Assignment), which seeks to minimize the network-wide interference by considering the three inter-dependent factors. Through extensive simulation, we show that PRACA significantly outperforms other schemes in terms of loss rate, delay, fairness and throughput.

I. INTRODUCTION

A Multi-radio Multi-channel Wireless Mesh Network (M-RMC WMN) is a multi-hop communication network made up of radio nodes equipped with multiple IEEE 802.11 standard radios to provide users Internet access. In an MRMC WMN, mesh nodes can transmit and receive packets simultaneously by communicating with their neighbours via a number of orthogonal frequency channels, thereof achieving higher system throughput than the traditional single-channel singleradio mesh network. Due to its promising performance, such multi-hop wireless networks have aroused much interest in both academia and the commercial sectors [1], [2].

In this paper, we study placement optimization problem for the deployment of a high performance MRMC WMN. We consider a general MRMC WMN as shown in Figure 1, which has an arbitrary number of Internet gateways (IGs) which connect directly to the Internet. There are a number of source routers (SRs) which are mesh routers (MRs) with end users. Routing is done in a multi-hop manner. An SR has a certain (heterogeneous) traffic requirement to either another destination router in the network or to the Internet (in which case the destination router can be any one of the gateways). Given the locations of IGs and SRs, we consider that there are some pre-defined *candidate* positions to place non-source mesh routers. A non-source router, without traffic requirement itself, helps forming routing paths by relaying traffic towards

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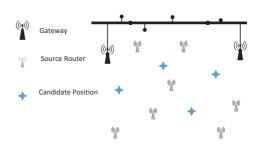


Fig. 1. A multi-radio multi-channel wireless mesh network with a number of candidate positions to place non-source routers.

any IGs. Each node in the multi-radio network is equipped with a possibly heterogeneous number of omni-directional radios. For simplicity and practical consideration, we will focus on single-path routing (instead of multipath routing).

The primary MRMC WMN deployment problem is to optimize where to place a given number of non-source mesh routers given their candidate positions. A poor placement of these routers leads to not only high interference level, but also unsatisfactory connectivity between mesh routers and gateways. Once the placement is determined, the performance of the network is largely affected by RCA (routing and channel assignment). Clearly, an effective channel assignment can make good spatial reuse of the orthogonal channels to reduce network interference. The routing decision is also important to reduce congestion in the network, leading to an overall improvement in system capacity.

The decisions of router placement, channel and routing assignment are inter-dependent because channel and routing assignment can only be designed based on the knowledge of router positions. On the other hand, it is not possible to optimize router placement with the knowledge of RCA. Therefore, these three parameters (router placement, channel assignment and routing) need to be optimized *jointly* at network deployment to achieve optimal performance. The problems we address is hence: Given the candidate positions and an available number of mesh routers, how to jointly select a subset of these positions for router placement, assign channels to each radio and route traffic for each source router?

Prior research on MRMC mesh optimization usually considers cases where the MR positions are known. Most other works on router placement has not considered sufficiently the RCA issue by using simplified objectives such as maximizing network connectivity, minimizing the number of deployed routers or maximizing coverage. The network topologies constructed hence may suffer from high interference. There has been little work on studying the *joint* optimization of placement and RCA.

The contributions of this work are as follows:

- Formulation of the joint optimization problem and its hardness analysis: We present the joint optimization problem for mesh deployment, which is to design router placement, routing and channel assignment to minimize interference for MRMC WMN, given heterogeneous traffic requirements of the source routers and number of radios in each router. We show that it is NP-hard.
- *PRACA: An Efficient Joint Algorithm for Router Placement and RCA:* To tackle the optimization problem, we propose a novel and simple heuristic called PRACA (Placement, Routing And Channel Assignment) to efficiently address the joint optimization problem.
- *Extensive Simulation Results:* Our scheme is evaluated through extensive simulation based on NS3. Simulation results show that PRACA substantially outperforms other schemes in terms of loss rate, throughput, delay and fairness.

We briefly discuss previous work as follows. Much of the literature tackles router placement without considering routing and channel assignment, while others consider placement with the objective of minimizing the number of routers used [3], [4]. As these works may lead to a network with congested routers, some schemes place traffic demand constraints on the router minimization problem [5]. There has also been placement work considering objective such as network connectivity and coverage (the number of existing routers covered by newly deployed routers) [6]. All the above has not considered routing performance and channel allocation. The setting we study is also different as the number of routers to be placed is given, instead of an optimizing parameter. Regarding routing and channel assignment, some have focused on the routing problem alone [7], while some others on channel assignment alone [8]. Though there are works on joint routing and channel assignment, they have not studied the router placement issue [9].

The rest of the paper is organized as follows. We first describe the system model, formulate the problem and analyze its complexity in Section II. We then present PRACA, an efficient heuristic for router placement, channel assignment and routing in Section III. In Section IV, we present illustrative simulation results with NS3. We conclude in Section V.

II. SYSTEM MODEL AND PROBLEM ANALYSIS

A. System Model

We consider that the locations of the gateways, sources and the maximum number of routers B (the budget for the number of routers) to be placed as relay nodes are known. We are also given candidate positions where we may place the B routers. Each mesh router is equipped with one or more 802.11 radios which can be tuned to any one of the orthogonal frequency channels. (The channels are orthogonal meaning transmissions on one channel do not interfere with those on another.) Each radio on a node will be assigned a unique channel, i.e., no two radios of the same node are assigned the same channel. (This is obvious because if two radios on a router are assigned the same channel only one can transmit at a time.) In order for two nodes to communicate with each other, both of them have to tune to the same common channel. The two routers also have to be within each other's communication range D, which is defined as the maximum transmission distance within which packets sent from a router can be successfully received by the intended receiver without other simultaneous transmission. Because frequent change of routing and channel assignment would introduce high control overhead and may lead to network disconnection, the routing and channel assignment are not expected to be updated frequently. Therefore, the RCA only needs to be calculated a few times a day.

We model the topology of the network (including the existing nodes and candidate positions) as a directed graph G(V, E), where V is the set of mesh nodes and E is the set of links in the network. A link (i, j) exists if router i and router j are in each other's communication range D. The set of neighbors of node i is denoted as N(i). Let $T \in V$ be the set of gateway routers, and $S \in V$ be the set of mesh routers. The set of candidate positions to place relay routers is denoted as $C = V \setminus (S \mid JT)$.

We consider heterogeneous number of radios at routers, the number of radios of router *i* is denoted as I_i . Traffic originates from source routers may be destined to any one of the gateways, or one of the routers in the network. Hence, each traffic flow requirement can be defined by (r; s(r); d(r); t(r)), where *r* is the index of the traffic requirement, d(r) is the destination, s(r) is the source and t(r) is the amount of traffic. Clearly, $s(r) \in S, d(r) \in V, \forall r$.

B. Interference Analysis

Interference is the major factor affecting the performance of a mesh network. Due to the broadcast nature of the wireless links, transmission along a communication link may interfere with transmissions along other communication links in the network. We consider the physical interference model to better capture interference in reality. Traffic demands between pairs of nodes in the mesh network are considered to be heterogeneous. The flow on link (i, j) suffers interference only if another link (i', j') with end points in the interference regions of the endpoints of link (i, j) has traffic flowing on it and is operating on the same channel as (i, j). The amount of interference suffered is proportional to the traffic on both links and the receive power from the source of the interfering link.

In general, receive power from j' to *i* decreases with transmission distance d(j', i), thus can be modeled as a function of transmit power and distance, i.e., $P_{j'}^i = \Theta(P_0, d(j', i))$. One of the commonly used model is the log distance path loss

model given by $P_{j'}^i = P_0 / (L_0 + 10n \log_{10} (d(j', i)/d_0))$, where d_0 is the reference distance, n is the path loss distance exponent and L_0 is the path loss reference distance.

C. Problem Formulation and Its Complexity

Let K be the set of orthogonal channels, R be the set of traffic requirement, and the boolean $x_{ij}^{r,k}$ indicate whether link (i, j) is used to relay traffic requirement r via channel k or not. Our objective is to minimize the network-wide interference defined as

$$\mathcal{I} = \sum_{k \in K} \sum_{i \in S} \sum_{i' \in S, i' \neq i} f_{i_{-}}^{k} f_{i'_{+}}^{k} P_{i'}^{i},$$
(1)

where $f_{i_{-}}^{k} = \sum_{r \in R} \sum_{j \in V} x_{ji}^{r,k} t(r)$ is the amount of traffic received at *i* via channel *k*, while $f_{i'_{+}}^{k} = \sum_{r \in R} \sum_{j \in V} x_{i'j}^{r,k} t(r)$ is the outgoing traffic from *i'* via channel *k*.

For any valid routing, we must have the following flow conservation constraints:

$$\sum_{j \in N(i)} \sum_{k \in K} x_{ij}^{r,k} - \sum_{j' \in N(i)} \sum_{k \in K} x_{j'i}^{r,k} = \begin{cases} 1, & \text{if } i = s(r); \\ -1, & \text{if } i = d(r); \\ 0, & \text{otherwise}; \end{cases}$$
(2)

 $\forall i \in V$. Define Z_i as the binary variable indicating whether there is router placed on position *i*. Binary variable Y_i^k indicates whether channel *k* is assigned to router *i*. A communication link (i, j) can be formed only if two router *i* and *j* are assigned with at less one common channel, i.e.,

$$X_{ij}^{r,k} \le Y_i^k + Y_j^k - 1, \forall i, j \in V \text{ and } r \in R.$$
(3)

We need to guarantee that there is a router placed on every relay node in G. A feasible router placement must satisfy the following constraints:

$$Y_i^k \le Z_i, \forall i \in V \text{ and } k \in K.$$
(4)

Due to its limited number of radios, router i must satisfy the following interface constraints:

$$\sum_{k} Y_i^k \le I_i, \forall i \in V.$$
(5)

It has been shown that jointly optimizing routing and channel assignment can result in a better solution than performing them sequentially or independently. Likewise, because the placement decision determines the network topology which constrains routing and channel assignment, we expect that *jointly* solving placement, routing and channel assignment leads to better performance than otherwise.

The channel assignment problem is known to be NP-hard as it can be reduced to the minimum edge coloring problem [10]. Therefore the joint placement, routing and channel assignment problem is also NP-hard because it can be reduced to the channel assignment problem when the router budget is zero and routing is done in advance.

III. PRACA: JOINT PLACEMENT, ROUTING AND CHANNEL ASSIGNMENT ALGORITHM

In this section, we present PRACA, an efficient joint optimization of placement, routing and channel assignment. We first discuss the RCA given router placement (Section III-A), followed by router placement given RCA (Section III-B). We end by remarking on the convergence and run-time complexity of PRACA (Section III-C).

A. Routing and Channel Assignment Given Router Placement

To address the routing and channel assignment sub-problem, we propose a local search algorithm based on Tabu search. The philosophy behind Tabu search is to search the solution space in several directions and change the current solution even when no neighbouring solution is better than the current solution. Tabu search keeps track of the best solution seen thus far and outputs this as the final solution.

• Algorithm overview: Each mesh router has a routing and channel assignment table. An example of routing and channel assignment table of router n is shown in Table I. Since there can be M = |S| - 1 + |T| (recall that T is the set of gateways and S is the set of mesh routers) possible destinations, there are M rows in the table. The first field of a row is the destination, the second field is the next-hop and the third field is the channel used for communication with next-hop. The solution to routing and channel assignment problem is represented by these tables. For example, the next-hop for destination i is j and the communication is on channel k. It means that the variable $X_{nj}^{i,k} = 1$ and $X_{nj'}^{i,k} = 0, \forall j' \neq j$ and $k \in K$.

Our algorithm starts with an initial solution and search among neighbour solutions in the attempt to reduce the interference in the successive iterations. Let f_m be the solution we get in iteration m. The search keeps a list of its most recently generated neighbours, known as a Tabu list \mathcal{L} . This prevents the search from repeating the same changes in a short space of time and so helps the algorithm to advance faster.

•*First Iteration*: To construct f_1 , we choose the shortest paths from router $i \in S$ to all its destinations as the initial routing paths for router *i*. The channel used by communication links are randomly chosen. The best solution f_{best} is set to be f_1 . • The m^{th} Iteration: The algorithm creates a neighbor from solution f_{m-1} as follows: It chooses a router n and a row ifrom n's table randomly. Then it change the next hop to j and the channel to k randomly. We check for routing loops when we assign the next hop and only accept a neighbour solution if it does not result in a loop. The neighbour is denoted as (n, i, j, k). In the *m* iteration, we create *h* neighbours such that each (n, i, j, k) is not in the tabu list \mathcal{L} . Whenever a new neighbour is created, it is added to \mathcal{L} . If the size of the Tabu List is greater than a specified threshold, the oldest members are removed from it. Function *interference()* is used to evaluate the fitness of a solution according to 1. We pick the neighboring solution with the lowest interference as f_m . We also keep track of the best solution seen so far.

Termination: We define U as the number of iterations through which the search can go without updating the best solution. If

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| | TABLE I | | |
|----------------|---------------|--------------|----------|
| ROUTING AND CH | IANNEL ASSIGN | MENT TABLE O | F ROUTER |
| | | | _ |

| Destination | Next-hop | Channel |
|--------------|----------|---------|
| i | j | k |
| : | : | : |
| i^{\prime} | j' | $k^{'}$ |

 f_{best} is not updated in U iterations, the algorithm terminates. The detail of the local search algorithm is shown in Algorithm 1 below.

Algorithm 1 LOCAL SEARCH ROUTING AND CHANNEL ASSIGNMENT

```
1: f_{best} \leftarrow f_1, \mathcal{L} \leftarrow null
2: m \leftarrow 0
3: while m \leq U do
       neighbours \leftarrow null
4:
       while neighbours.size() \le h do
5:
6:
          create a neighbour f_{neighbour}
           if f_{neighbour} \notin \mathcal{L} then
 7.
              add f_{neighbour} to both neighbours and \mathcal{L}
8:
           end if
9٠
       end while
10:
       f_m \leftarrow bestNeighbour(neighbours)
11:
       if interference(f_m) < interference(f_{best}) then
12:
13:
           f_{best} \leftarrow f_m
           m \leftarrow 0
14:
       end if
15:
16: end while
```

When we run Tabu Search, we assign channels to links and not to radios themselves. Therefore the solution we get may have cases where the number of channels assigned to a node is greater than the number of radios on the node. We therefore have to remove some channels, using a pruning algorithm shown in Algorithm 2.

To do that, we first picks the router u where the number of channels most exceeds the number of the actual radios (Line 2 in Algorithm 2). As we want to reduce the number of distinct channel assigned to u, we need to merge two channels. This will increase the interference. We hence choose two channels i and j that are assigned to u and their merging will minimize the increase in interference (Line 3 in Algorithm 2). Then, we change the channel of links incident to node u from j to i (Line 4-7). The change may increase the number of channel assigned to the neighbours of u. For example, setting $x_{u,v}^i = 1$ is equivalent to assigning channel i to v if channel i is not assigned to v originally. Therefore the change may propagate to u's neighbours. If the channel constraint is violated at node v after the change, we can recursively call Algorithm 2 to prune a channel. Otherwise, we do not propagate the change.

Figure 2 shows how the pruning algorithm works. Assuming there are two radios at each router. In Figure 2(a), router b is assigned with 3 different channels. We hence merge channels

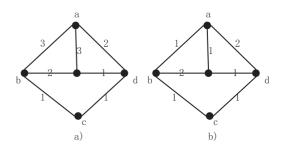


Fig. 2. An example of pruning operation.

by switching channel 3 to channel 1. However, this will assign one more channel to router a violating the interface constraint at router a. Therefore, the pruning is propagate to router a. Router a change all link operating on channel 3 to 1. The resultant network is shown in Figure 2(b).

| Algorithm 2 PRUNING ALGORITHM | | |
|-------------------------------|--|--|
| 1: | while $\exists x : \sum_k Y_x^k > I_x$ do | |
| 2: | Select $u: \sum_{k} Y_{u}^{k} - I_{u} \ge \sum_{k} Y_{x}^{k} - I_{x}, \forall x$ | |
| 3: | Select channel i and j such that both channel i and j | |
| | are assigned to u and switching links on channel j to | |
| | channel i will minimize the increase in interference | |
| 4: | for each $x_{u,v}^j = 1$ or $v: x_{v,u}^j = 1$ do | |
| 5: | $x_{u,v}^j \leftarrow 0, x_{u,v}^i \leftarrow 1, x_{v,u}^j \leftarrow 0, x_{v,u}^i \leftarrow 1$ | |
| 6: | end for | |
| 7: | end while | |
| | | |

B. Router Placement Given RCA

The placement algorithm jointly considers routing and channel assignment decision while choosing positions to place routers. We first consider the case where we have unlimited number of mesh routers to place. Based on this placement, we calculate a RCA (routing and channel assignment) solution. Then, we greedily remove one router with the least traffic flow (we hence reduce the number of deployed router by 1). We keep iterating until we are within the specified budget (at most *B* routers are deployed).

We define an **active** router as a mesh router with traffic flowing through it. Amongst these, there are some routers which are essential in maintaining connectivity. A **non-critical** router is one whose removal does not result in any source becoming disconnected from the gateways. As postprocessing, we will prune inactive routers that are non-critical.

C. Convergence and Run-time Complexity

The solution gets closer to the optimum as we go through more iterations of Tabu search. The number of neighbours created in each iteration and the size of the Tabu list are key parameters in PRACA. A larger Tabu list allows a wider space to be explored and so does increasing the number of neighbours.

Define h as the number of links in a network and n as the number of nodes. Creating each neighbour takes $O(h^2)$ operations since we have to recompute the traffic on each link. Once the neighbours are created, evaluating the objective on each neighbour takes $O(h^2)$. Each iteration of PRACA takes $O(h^4)$ time.

The number of link h is bound by n^2 (i.e. In a complete connected graph, we have $h = n^2$). In reality, the mesh network is much sparser, and hence h is much smaller than n^2 . Therefore, the time complexity of PRACA is polynomial in the number of links.

IV. ILLUSTRATIVE SIMULATION RESULTS

A. Simulation Environment and Metrics

We have implemented PRACA using C++ and NS3 to study its performance. In our simulations, existing routers are randomly distributed in a 400m \times 400m area. Similarly, candidate locations for placement are also randomly selected in the area whilst gateways are placed close to the center of the area. We consider all traffic requirements are going to the gateways, though our algorithm can be applied to traffic destined to any router.

We use the log distance loss model in NS3 to model signal fading with distance. For both UDP and TCP, we inject traffic continuously in the network according to a given demand. We evaluate the performance in terms of loss rate, throughput, delay and *Jain's fairness index* in the throughput of different flows. *Jain's fairness index* can be calculated as $(\sum_{i=1}^{n} x_i)^2/(n \sum_{i=1}^{n} x_i^2)$, where x_i is the throughput of the *i*th source nodes. Clearly the index ranges from 1/n to 1, and it is maximum when all source nodes achieved the same throughput.

Since PRACA is the first work considering the joint design of placement, routing and channel assignment, we compare PRACA with a composed scheme labeled as GBP (greedy algorithm based placement). This scheme uses a greedy algorithm to select a set of positions that maximize the number of paths between the source routers and any one of the gateway while guaranteeing the connectivity of the network. GBP uses a channel assignment scheme [11] based on Tabu Search. We use Short Path First (SPF) for routing in GBP. Unless otherwise stated, we use the following baseline parameters: IEEE 802.11a Wi-Fi standard, communication range (meters) D = 100, number of radios in each router $I_i = 3$, number of orthogonal channels |K| = 5, number of sources |R| = 5, traffic demand per flow (Mbps) t(r) = 8, number of existing routers in the network $\alpha = 15$, number of candidate positions $\beta = 40$, placement budget $\gamma = 10$.

B. Illustrative Results

In Figure 3 we see how the UDP loss rate varies with increasing demand. The loss rate increases due to increased interference. PRACA greatly outperforms GBP. This is because in Routing and Channel Assignment, PRACA considers flow traffic and hence optimizes the routing, and especially channel assignment such that links with a lot of traffic on them get the least used channels. The second factor which makes PRACA perform well is that PRACA jointly optimizes

RCA with router placement, which will select positions that is beneficial to routing performance. Whereas in GBP, only the connectivity of the network is considered. Therefore in the network constructed by GBP, there may be cases that traffic from source routers need to travel long routing paths to the gateways as routing performance is not considered. The third factor is that in GBP only interference between links within each other's communication range is captured, in PRACA interference between any two links is considered by the model.

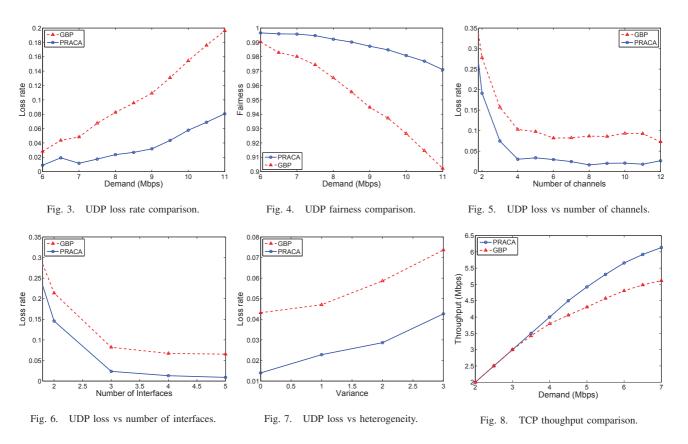
The Jain's fairness index for UDP throughput is shown in Figure 4. Fairness decreases with traffic rate, because flows are completing for the limited resources in the network. As the traffic rate increases, some flows start to receive unfair allocation. PRACA achieves much better and more stable fairness than the other scheme, because routing performance are jointly considered by PRACA. However in GBP, performance is not considered. Some relay nodes may be used to relayed traffic for many source routers. Resources are not fairly allocated in GBP.

In Figure 5 we plot UDP loss rate versus the number of orthogonal channels. As we increase the channels, both PRACA and GBP respond by using up the additional channels to minimize the number of interfering links hence we observe an improvement in performance. The substantially lower loss rate of PRACA means that it is able to make better use of channels. As the loss rate flats off when the number of channels is similar to the number of antennas, it implies that there is little incremental benefit in having channels much more than the number of radios.

In Figure 6 we plot UDP loss rate verus the number of interfaces (radios) on each router. The curves first drop sharply and then more slowly. Our results reveal that once we have a certain number of interfaces on each node. Once the number is beyond a few (say 3), adding more interfaces does not result in significant improvement in performance. The trend remains the same even when we have a greater number of channels in the network.

In Figure 7 we explore the effect of heterogeneity in the number of interfaces. The number of interfaces on each router is generated according to uniform distribution. With the mean number of interfaces remains the same as our baseline (3), we increase the variance. Loss rate increases with increasing variance. This is because routers with the least number of interfaces will become bottlenecks of the topology. This result is consistent with Figure 6 which tells us that beyond a certain point, we get no much marginal benefit by adding more interfaces. It is better to evenly distribute the interfaces amongst the radios.

Lastly, we show in Figure 8 TCP throughput achieved by PRACA and GBP versus traffic demand. While the network throughput increases quite linearly with demand in PRACA, the throughput of GBP flats off much earlier with much lower network capacity. This is because PRACA maximizes the benefit of relay routers by selecting good channels to reduce interference and balancing traffic by routing. In contrast, GBP always maximizes the number of paths between sources



and gateways without awareness of traffic demand, channel utilization and the quality of routing paths.

V. CONCLUSION

In this paper we have formulated and addressed the joint optimization of router placement, routing and channel assignment for multi-radio multi-channel wireless mesh deployment. The network has multiple gateways and a number of candidate positions to place a certain number of routers. Our objective to to minimize network interference, so as to achieve good network performance.

We consider an interference objective which takes into account the expected traffic on links and the distance between nodes. The optimization problem is shown to be NP-hard. We present PRACA, an efficient algorithm for the joint problem. PRACA is driven by Tabu search, a well known local search method using our interference objective. We study PRACA with NS3 simulation. Our results show that it significantly outperforms other schemes in terms of loss rate, delay, fairness and throughput.

REFERENCES

- T. Chen and S. Zhong, "Perfectly fair channel assignment in noncooperative multi-radio multi-channel wireless networks," *Computer Communications*, vol. 32, pp. 1058–1061, Apr. 2009.
- [2] X. Wu, S.-H. Chan, and B. Mukherjee, "MADF: A novel approach to add an ad-hoc overlay on a fixed cellular infrastructure," in *Wireless Communications and Networking Confernce, 2000. WCNC. 2000 IEEE*, vol. 2. IEEE, 2000, pp. 549–554.

- [3] J. Wang, B. Xie, K. Cai, and D. P. Agrawal, "Efficient mesh router placement in wireless mesh networks," in *Mobile Adhoc and Sensor Systems*, 2007. *MASS 2007. IEEE Internatonal Conference on*. IEEE, 2007, pp. 1–9.
- [4] M. Ahlberg, V. Vlassov, and T. Yasui, "Router placement in wireless sensor networks," in *Mobile Adhoc and Sensor Systems (MASS)*, 2006 *IEEE International Conference on*. IEEE, 2006, pp. 538–541.
- [5] J. Wang, K. Cai, and D. R. Agrawal, "A multi-rate based router placement scheme for wireless mesh networks," in *Mobile Adhoc and Sensor Systems, 2009. MASS'09. IEEE 6th International Conference on*. IEEE, 2009, pp. 100–109.
- [6] T.-W. Kuo, K. C.-J. Lin, M.-J. Tsai, W. Shen, C. Hsu, C. Chou, M. Yang, Y. Wang, C.-C. Cheng, P.-C. Hsiu *et al.*, "Maximizing submodular set function with connectivity constraint: Theory and application to networks," in *IEEE INFOCOM*, 2013.
- [7] G. Jakllari, S. Eidenbenz, N. Hengartner, S. V. Krishnamurthy, and M. Faloutsos, "Link positions matter: A noncommutative routing metric for wireless mesh networks," *Mobile Computing, IEEE Transactions on*, vol. 11, no. 1, pp. 61–72, 2012.
- [8] X. Yue, C.-F. Wong, and S.-H. Chan, "CACAO: Distributed clientassisted channel assignment optimization for uncoordinated WLANs," *Parallel and Distributed Systems, IEEE Transactions on*, vol. 22, no. 9, pp. 1433–1440, 2011.
- [9] P. Dutta, V. Mhatre, D. Panigrahi, and R. Rastogi, "Joint routing and scheduling in multi-hop wireless networks with directional antennas," in *INFOCOM*, 2010 Proceedings IEEE. IEEE, 2010, pp. 1–5.
- [10] M. K. Marina and S. R. Das, "A topology control approach for utilizing multiple channels in multi-radio wireless mesh networks," in 2nd International Conference on Broadband Networks, vol. 1, 2005, pp. 381–390.
- [11] A. P. Subramanian, H. Gupta, and S. R. Das, "Minimum interference channel assignment in multi-radio wireless mesh networks," in *Sensor*, *Mesh and Ad Hoc Communications and Networks*, 2007. SECON'07. 4th Annual IEEE Communications Society Conference on. IEEE, 2007, pp. 481–490.