Recurrence Relations on Transfer Matrices Yield Good Lower and Upper Bounds on the Channel Capacity of Some 2-Dimensional Constrained Systems (Extended Abstract)*

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There has been much recent research into calculating the capacities, of 2-dimensional constrained systems (binary matrices). A *constraint* is a rule that specifies which binary matrices are legal. The *capacity* of the constraint is $C = \lim_{k,r\to\infty} \frac{\log_2 f(k,r)}{kr}$, where $f(k,r)$ is the number of $r \times k$ legal binary matrices.

Almost all current work on calculating capacity uses the, by now ubiquitous, *transfer matrix* approach. Given a particular 2-dimensional constraint the transfer matrix approach essentially works by defining an infinite sequence, A_n, $n = 1, 2, 3, \ldots$ of larger and larger matrices that have the property that $C = \lim_{n\to\infty} \frac{\log_2 \lambda_n}{n}$ where λ_n is the largest eigenvalue of A_n.

In practice, closed formulas for capacities can not be found and only upper and lower bounds are derivable. These are found by calculating as many λ_n as possible, plugging these values into some standard inequalities and taking the best upper and lower bounds that appear. A major difficulty with this approach is that the A_n usually grow exponentially in n so it is often only possible to find the first few values of λ_n.

One often noted fact is that, for almost every constraint studied, the A_n satisfy a recurrence relation. Surprisingly, this fact has not proven useful in deriving good upper and lower bounds on capacity.

In this poster we discuss two classes of constrained systems; generalizations of (i) Read/Write Isolated Memory and (ii) two-dimensional $(1, \infty)$ run-length limited constrained systems, and show how to use the recurrence relations on the A_n and '1'-counting to derive recurrence inequalities on the λ_n that yield good upper and lower bounds on the capacities of the constraints. We observe that, in contrast to the situation in most other known constraints, this technique provides much better bounds than the simple brute force method.

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