

# Efficient Influence Maximization in Social Networks

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*Wei Chen, et al, "Efficient Influence Maximization in Social Networks", KDD09'*

# OUTLINE

- Problem
- Previous Work
- Degree Discount Heuristics
- Summary
- References

# Problem Statement

- Find a small subset of nodes in a social network that could maximize the spread of influences.
- Known as *Influence Maximization*
- A.k.a *Viral Marketing* which makes use of “word-of-mouth marketing” properties of social network

**Viral Marketing**



# Problem Statement

- Optimization problem first introduced by Domingos and Richardson, KDD01'/02', *NP-hard to solve*
- Elegant graph formulation introduced by Kempe, et al, KDD03'

Given:

- ✓ A graph  $G(V, E)$ :
  - Vertices: individuals in social network
  - Edges: connection or relationship
- ✓  $k$ , size of output seeds
- ✓ A cascade model: LTM, ICM

Output:

$S$ , a set of seeds (nodes) that maximize the expected number of nodes active in the end

# Problem Statement: Cascade Model



- Models how influences propagate
- Linear Threshold Model (*LTM*)
- Independent Cascade Model (*ICM*)
- ... ..
- Analogous to Epidemic Models like SIS, SIR

# Linear Threshold Model

- A node  $u$  has random threshold  $\theta_u \sim U[0, 1]$
- A node  $u$  is influenced by each neighbor  $v$  according to a *weight*  $b_{uv}$  which satisfies:

$$\sum_{v \text{ neighbor of } u} b_{u,v} = 1$$

- A node  $u$  becomes active when at least  $\theta_u$  fraction of its neighbors are active

$$\sum_{v \text{ active neighbor of } u} b_{u,v} \geq \theta_u$$

# Independent Cascade Model



- When node  $u$  becomes active, it has a *single* chance of activating each currently inactive neighbor  $v$ .
- The activation attempt succeeds with probability  $p_{uv}$ .
- In both LTM and ICM, active nodes never deactivate.

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## Previous Work:

### “Maximizing the Spread of Influence Through a Social Network”, KDD03’



- Proposed by D.Kempe, J.Kleinberg and E.Tardos
- Greedy hill-climbing algorithm:

In each round add a vertex  $v^*$  into  $S$  such that  $v^*$  and  $S$  maximize the influence spread  $f$ :

$$v^* = \arg \max_v f(S + v) - f(S)$$

- Monte Carlo:

Influence spread is estimated with  $R$  repeated simulations

- Effectiveness:

Can guarantee a solution with  $(1 - 1/e)$  of the optimal

- Drawback:

poor efficiency, 15,000 nodes takes a few days to compute

## Previous Work:

### “Cost-effective Outbreak Detection in Networks”, KDD07’

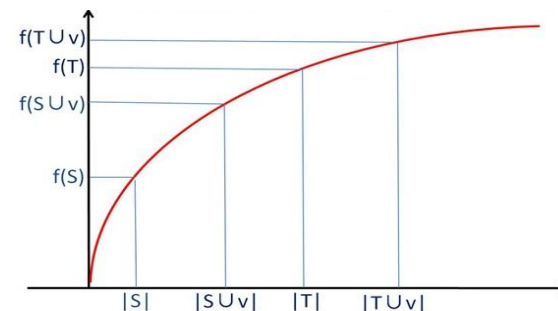
- Proposed by J. Leskovec, A. Krause, et al

- Cost-effective Lazy Forward algorithm:

The CELF optimization utilizes submodularity of influence spread function to greatly reduce the number of evaluations of vertices, and get the same performance as the original greedy algorithm.

- Submodularity:

$$\forall S \subset T \subset N, \forall v \in N \setminus T,$$
$$f(S + v) - f(S) \geq f(T + v) - f(T)$$



- Efficiency:

approximately 700 times fast than original greedy algorithm, but still hours to finish.

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# Degree Discount Heuristics



- Proposed by W.Chen, Y.Wang , S.Yang from MSRA and Tsinghua
- High Efficiency:

Amazingly reduces the running time by over *six orders* of magnitude with *less than 3.5%* degradation in performance.
- Motivation:

Conventional degree/centrality based heuristics perform poorly in practical scenarios because they *ignore the network effect*.

Important Fact: Since many of the most central nodes may be clustered, targeting all of them is not at all necessary.

# Degree Discount Heuristics

- **Basic Idea**

Consider edge  $\overline{uv}$ , with  $u$  in the seed set  $S$  and  $v$  being considered. Since  $u$  is in the seed set, by taking network effect into consideration, we should not count edge  $\overline{uv}$  towards  $v$ 's degree. i.e. Degree Discount

- **Assumption**

In ICM, when propagation probability  $p$  is small, we may ignore indirect influence of  $v$  to multi-hop neighbors and focus on the *direct influence* of  $v$  to its immediate neighbors.

**Remarks** : Is this assumption still reasonable when  $k$  is small ? Or when neighbor overlapping is prominent?

# Degree Discount Heuristics

- Degree Discount Model:

$t_v$  -- number of  $v$ 's neighbors that in seed set  $S$

$d_v$  -- degree of node  $v$

- ✓ Probability that  $v$  is influenced by its immediate neighbors:  $1 - (1 - p)^{t_v}$   
in such case, selecting  $v$  does not contribute additional influence.
- ✓ Probability that  $v$  is not influenced by its immediate neighbors:  $(1 - p)^{t_v}$   
in such case, selecting  $v$  will in expectation influence  $1 + (d_v - t_v) * p$  vertices.

So that the *expected number of additional vertices influenced by selecting  $v$  as seed* is:

$$\begin{aligned} & [1 - (1 - p)^{t_v}] * 0 + [(1 - p)^{t_v}] * [1 + (d_v - t_v) * p] \\ &= (1 - t_v * p + o(p)) * (1 + (d_v - t_v) * p) \\ &\cong 1 + (d_v - 2t_v - (d_v - t_v) * t_v * p) * p \triangleq A \end{aligned}$$

If no neighbor of  $v$  is selected as seed, the answer above is  $1 + d_v * p \triangleq B$

Let  $\gamma$  be the degree discount caused by each neighbor in seed set, then

$$\begin{aligned} \gamma * t_v * p &= B - A \\ \gamma &= 2 + (d_v - t_v) * p \end{aligned}$$

# Degree Discount Heuristics

- Algorithm:

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**Algorithm 4** DegreeDiscountIC( $G, k$ )

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```
1: initialize  $S = \emptyset$ 
2: for each vertex  $v$  do
3:   compute its degree  $d_v$ 
4:    $dd_v = d_v$ 
5:   initialize  $t_v$  to 0
6: end for
7: for  $i = 1$  to  $k$  do
8:   select  $u = \arg \max_v \{dd_v \mid v \in V \setminus S\}$ 
9:    $S = S \cup \{u\}$ 
10:  for each neighbor  $v$  of  $u$  and  $v \in V \setminus S$  do
11:     $t_v = t_v + 1$ 
12:     $dd_v = d_v - 2t_v - (d_v - t_v)t_v p$ 
13:  end for
14: end for
15: output  $S$ 
```

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# Degree Discount Heuristics

- Evaluations on NetHEPT:

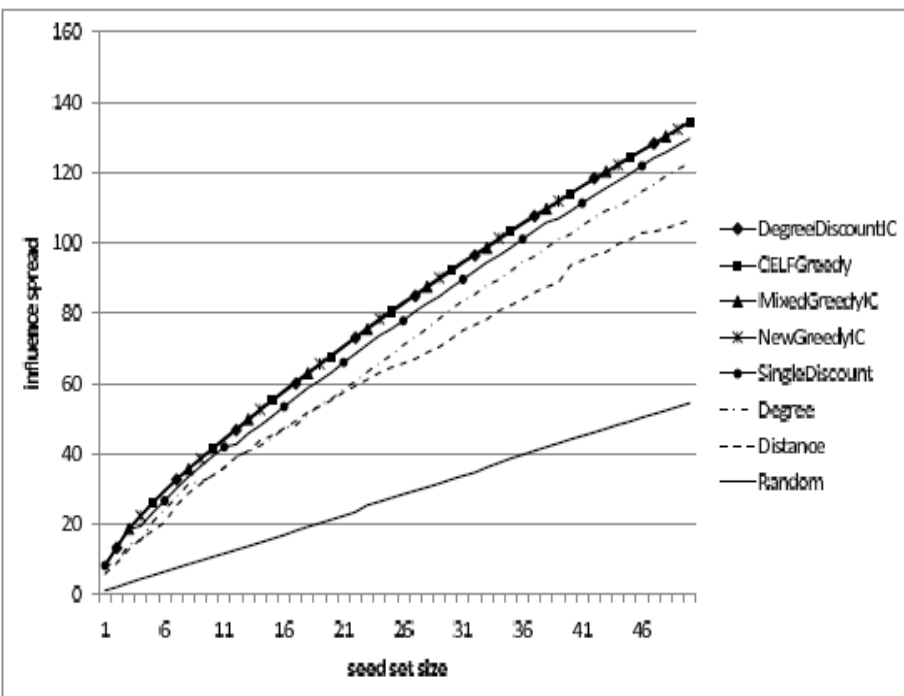


Figure 1: Influence spreads of different algorithms on the collaboration graph NetHEPT under the independent cascade model ( $n = 15,233$ ,  $m = 58,891$ , and  $p = 0.01$ ).

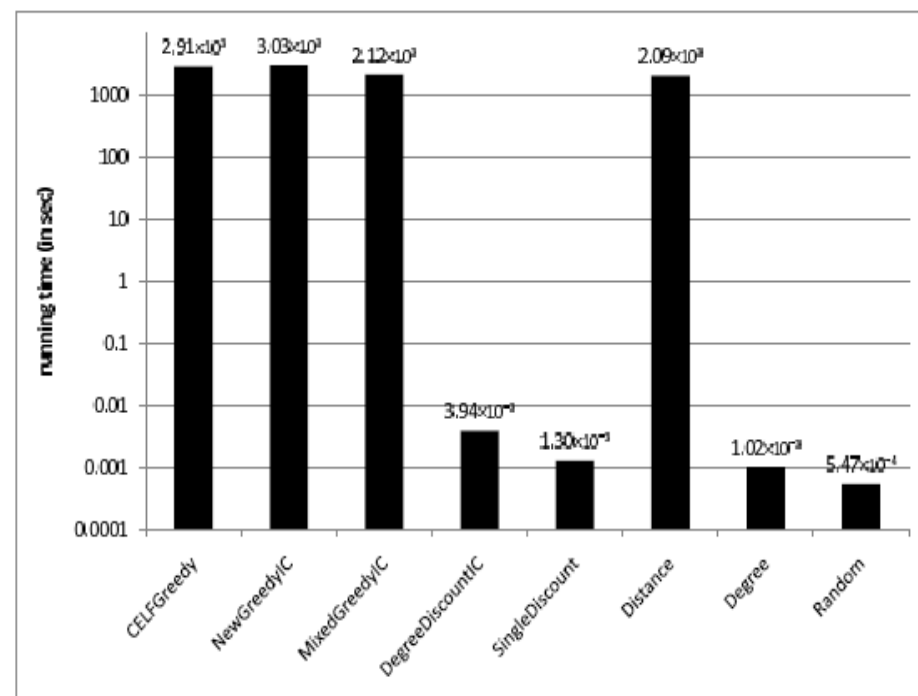


Figure 3: Running times of different algorithms on the collaboration graph NetHEPT under the independent cascade model ( $n = 15,233$ ,  $m = 58,891$ ,  $p = 0.01$ , and  $k = 50$ ).



# Degree Discount Heuristics

- Evaluations on NetPHY:

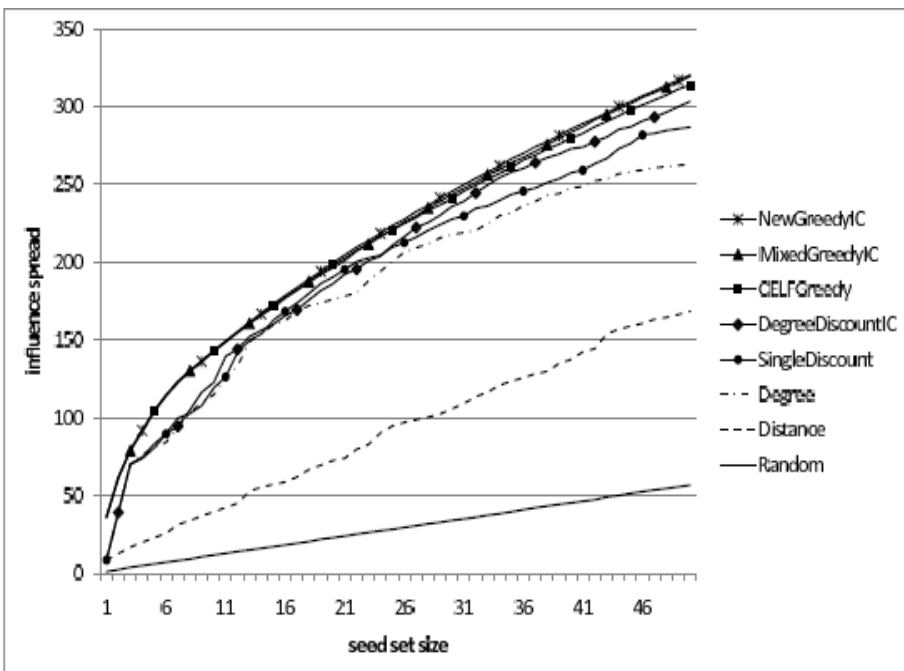


Figure 2: Influence spreads of different algorithms on the collaboration graph NetPHY under the independent cascade model ( $n = 37,154$ ,  $m = 231,584$ , and  $p = 0.01$ ).

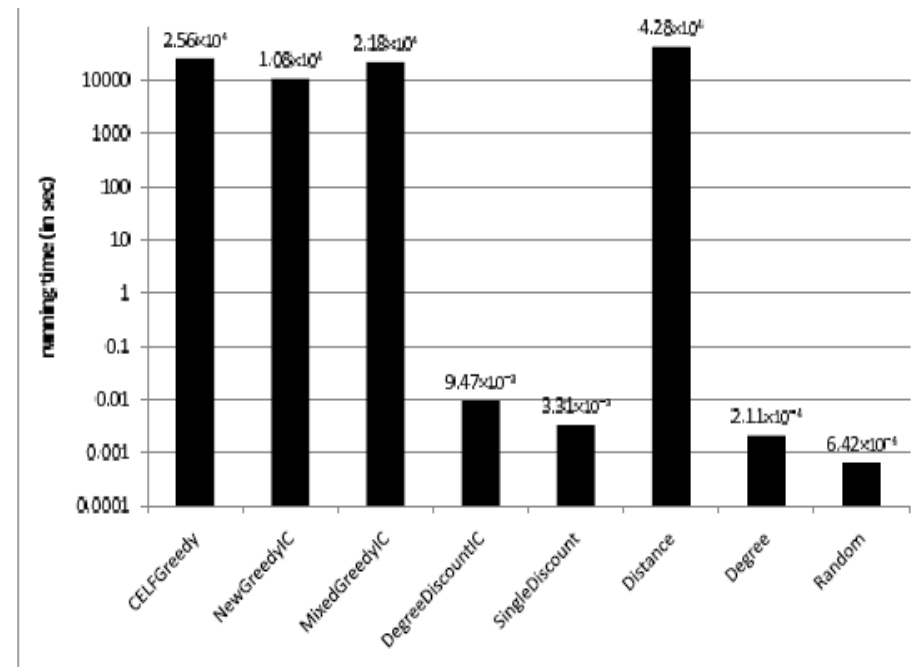


Figure 4: Running times of different algorithms on the collaboration graph NetPHY under the independent cascade model ( $n = 37,154$ ,  $m = 231,584$ ,  $p = 0.01$ , and  $k = 50$ ).

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# Summary



- The current influence maximization problem is simplified, without considering other features in social networks, such as community structures and small-world phenomenon.
- The author suggests that we should focus our research efforts on searching for more effective heuristics for different influence cascade model in real life influence maximization applications
- More sophisticated heuristics are promising, such as taking into consideration multiple links between nodes, higher-order influences, cross-neighborhood structure...

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- *W. Chen, Y. Wang and S. Yang , “Efficient Influence Maximization in Social Networks”, KDD 2009*
- *D. Kempe, J. Kleinberg and E. Tardos, “Maximizing the Spread of Influence through a Social Network”, KDD 2003*



Thank you !