Statistical Analysis of Background Noise in Diffusion Images

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Introduction: The analysis of the signal-to-noise ratio (SNR) in diffusion images requires a good understanding of background noise characteristics. In this study, we analytically derived and validated the mean and standard deviation (SD) of the background noise in calculated diffusion maps.

Theory and Methods: Let S_0 and S_1 be the random variables representing the intensity values of the background voxels in the baseline spin-echo (SE) image (b=0) and the diffusion-weighted image respectively. We assume that S_0 and S_1 are corrupted by Rayleigh noise with SDs σ_0 and σ_1 respectively [1]. Let D be a random variable representing the diffusion coefficient, which is calculated using the division and natural logarithm operators, such that $D = (1/b) \cdot \log(S_0/S_1)$. These two highly non-linear image-processing operators have been shown qualitatively to alter the shape of the noise distribution [2]. In order to develop a better insight into the noise properties of calculated diffusion maps, we derived the probability density function of D, $f(D)$. This takes the form $f(D) = 2\gamma b \cdot e^{2bD} \big/ \left(e^{2bD} + \gamma\right)^2$, where $\gamma = \sigma_0^2 / \sigma_1^2$ and the mean μ_D and SD σ_D of $f(D)$ were found to be $(1/b) \cdot \log(\sigma_0/\sigma_1)$ and $\pi \big/ \left(2\sqrt{3}b\right)$, respectively.

Results and Discussion: Monte Carlo simulations of a diffusion experiment were used to validate the theoretical mean and SD. The b-factor was varied from 500 smm^{-2} to 1500 smm^{-2} (200 smm^{-2} step) and the ratio σ_0/σ_1 varied from 0.01 to 100. For each b-factor and ratio σ_0/σ_1, a 256×256-pixel baseline SE image S_0 and a diffusion-weighted image S_1 were simulated and D was then calculated pixel-by-pixel. The mean and SD of D were computed, and compared with the theoretical mean and SD and were well matched, as can be seen from Figs. 1 & 2. The discrepancy, which was defined as the root-mean-squared (RMS) error, was $0.011\times10^{-3}\,mm^2s^{-1}$ for the mean and $0.009\times10^{-3}\,mm^2s^{-1}$ for the SD. The analytical solutions for the mean and SD were further validated using clinical data. Fifty pairs of baseline SE and diffusion-weighted images (b ≈ 700 smm^{-2}), obtained from 10 human subjects, were used for validation. Regions of interest (ROIs) were drawn manually on each pair of images to enclose the background noise regions. In practice the SDs σ_0 and σ_1 can be estimated by $\sqrt{2/\pi} \cdot E[i]$, where $E[i]$ is the average intensity value inside the ROI. As shown in Figs. 3 & 4, the discrepancy is negligibly small, particularly when allowing for the effects of residual image ghosting in the noise measurements. (RMS errors: $0.024\times10^{-3}\,mm^2s^{-1}$ for mean and $0.043\times10^{-3}\,mm^2s^{-1}$ for SD).

Monte Carlo Simulation

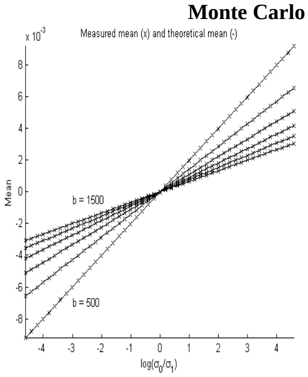

Fig. 1: Mean

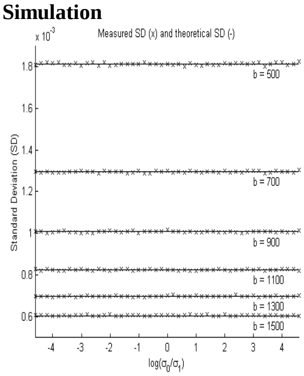

Fig. 2: Standard deviation

Clinical Validation

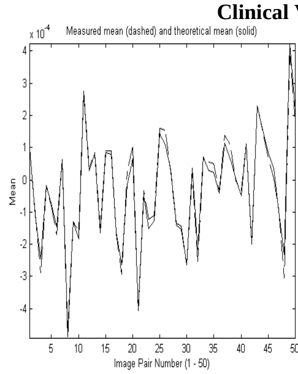

Fig. 3: Mean

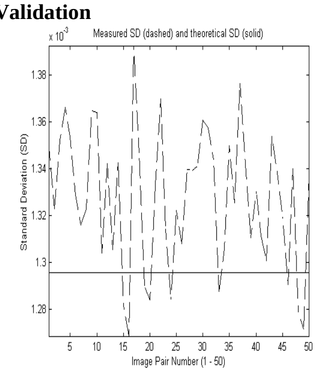

Fig. 4: Standard deviation

Conclusion: Analytical solutions for the mean and SD of the background noise in maps of D were derived and validated using Monte Carlo simulations and clinical data. The results show that the theoretical mean and SD and the measured mean and SD were well matched.

References: 1. Andersen AH and Kirsch JE, Medical Physics 23 (6), 1996.
2. Summers P, Chung ACS and Noble JA, Abstract 1779, ISMRM 2000.