# Tracking Distributed Data

# Ke Yi HKUST

#### The Distributed Count-Down Problem



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Naive solution: O(n) communication

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- When one local count reaches n/k, broadcast to
  - Compute the current total count
  - Compute new leeway = n total count
  - Set new threshold = leeway / k

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## The Count-Tracking Problem



Counters increment over time

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Sends a message when  $n_i$  reaches a threshold



Relative  $\varepsilon$ -error for each  $n_i$ 



Sends a message when  $n_i$  reaches a threshold



Communication is one-way

### Deterministic Lower Bound

#### Theorem

Any deterministic protocol that solves the count-tracking problem must communicate  $\Omega(k/\varepsilon \cdot \log n)$  messages, even with two-way commucation.

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Sends *n<sub>i</sub>* with probability *p* when a new item arrives









$$\hat{n}_i = \left\{ egin{array}{cc} ar{n}_i - 1 + 1/p, & ext{if } ar{n}_i ext{ exists;} \ 0, & ext{else.} \end{array} 
ight.$$

$$E[\hat{n}_i] = n_i$$
,  $Var[\hat{n}_i] = 1/p^2$ 



$$\hat{n}_{i} = \begin{cases} \bar{n}_{i} - 1 + 1/p, & \text{if } \bar{n}_{i} \text{ exists;} \\ 0, & \text{else.} \end{cases}$$
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$$\hat{n} = \sum \hat{n}_{i}$$

$$\mathsf{E}[\hat{n}] = \sum \hat{n}_i = n$$
,  $\mathsf{Var}[\hat{n}] = k/p^2$ 

#### Rounds

#### Chebyshev inequality

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#### Chebyshev inequality

SD less than  $\varepsilon n \to p = O(\sqrt{k}/\varepsilon n)$  constant probability of success (at any one time instance)

- Track a 2-approximation n
   of n
   using the deterministic
   algorithm
  - Broadcast  $\bar{n}$  whenever  $\bar{n}$  doubles

• Set 
$$p = \frac{\sqrt{k}}{2\overline{n}}$$

- Divide the tracking period into rounds
  - n changes by at most a constant factor in a round
  - *p* is fixed in a round

#### Communication Cost

- Communication cost
  - Tracking a 2-approximation:  $O(k \log n)$
  - Number of messages in a round:  $O(np) = O(\sqrt{k}/\varepsilon)$
  - Total:  $O(k \log n + \sqrt{k}/\varepsilon \cdot \log n)$ 
    - Can be improved to  $O(k \log n / \log(k\varepsilon^2) + \sqrt{k} / \varepsilon \cdot \log n)$

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- Lower bounds
  - Only allow one-way communication:  $\Omega(k/\varepsilon \cdot \log n)$ (randomization doesn't help)

• Two-way communication:  $\Omega(k + \sqrt{k}/\varepsilon \cdot \log n)$ 

### Tight Bounds for Count-Tracking

- Upper bound in words
- Lower bound in number of messages

$$k < 1/\varepsilon^2$$
 $k > 1/\varepsilon^2$  $\Theta(\sqrt{k}/\varepsilon \cdot \log n)$  $\Theta\left(k \frac{\log n}{\log(k\varepsilon^2)}\right)$ 

[Huang, Yi, Zhang, PODS'12]

#### The Distributed Streaming Model


# The Distributed Streaming Model





Communication model (One-shot model)



Communication model (One-shot model)

Data stream model



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Communication model (One-shot model)

Data stream model

- The count-down problem
- Count-tracking
- Frequent items (heavy hitters)
- Random sampling
- Other problems









Estimate the frequency of every element with additive error  $\varepsilon n$ .

Use the previous algorithm on each item *i* 

- Maintain a count for each item at each site
- Space

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Space

Streaming algorithm (Misra-Gries) cost per site:  $O(1/\varepsilon)$ 

- total:  $O(k/\varepsilon)$
- improve to  $O(\sqrt{k}/\varepsilon)$

## Frequent Items: Algorithm

Idea: maintain only large enough counts

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# *i*: •• • •• •

Start to count i withUpdate the countprobability pwith probability p



Coordinator only know  $\bar{c}$ 



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$$\hat{f}_i = \begin{cases} \bar{c} - 1 + 2/p, & \text{if } \bar{c} > 0; \\ 0, & \text{else.} \end{cases}$$



Coordinator only know  $\overline{c}$ 

$$\hat{f}_i = \begin{cases} \bar{c} - 1 + 2/p, & \text{if } \bar{c} > 0; \\ 0, & \text{else.} \end{cases}$$

Bias might be as large as  $\varepsilon n/\sqrt{k}$ 



Coordinator only know  $\bar{c}$ 



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$$\hat{f}_i = \begin{cases} c-1+1/p, & \text{if } c > 0; \\ 0, & \text{else.} \end{cases}$$



#### Estimate c by $\overline{c}$

$$\hat{c} = \left\{ egin{array}{c} ar{c} - 1 + 1/p, & ext{if } ar{c} > 0; \\ 0, & ext{else.} \end{array} 
ight.$$



Combined estimator

$$\hat{f}_i = \begin{cases} \bar{c} - 2 + 2/p, & \text{if } \bar{c} \ge 2; \\ 1/p, & \text{if } \bar{c} = 1; \\ 0, & \text{else.} \end{cases}$$

• 
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set  $p = O(\frac{\sqrt{k}}{\varepsilon n})$ space:  $O(\sqrt{k}/\varepsilon)$ space per site:  $O(1/(\varepsilon \sqrt{k}))$ communication: same as before

- Communication lower bound still hold
- Space lower bound

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  - Communication-space tradeoff

#### Theorem

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Communication cost:  $O(\sqrt{k}/\varepsilon \cdot \log n)$  bits Space per site:  $\Omega(1/(\varepsilon\sqrt{k}))$  bits

# Communication Complexity

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The k-party communication complexity for the one-shot frequency estimation problem is  $\Omega(\sqrt{k}/\varepsilon)$  bits.

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#### Direct-Sum theorem

Solve  $\ell$  instances of the frequency estimation problem simultaneously needs  $\Omega(\ell \cdot \sqrt{k}/\varepsilon)$  bits of communication.

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Proof sketch

Let  $\mathcal{A}$  be a k-party tracking algorithm with communication C and space M

Use  $\mathcal{A}$  to solve *tk*-party one-shot problem.

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#### Reservoir Sampling [Waterman '??; Vitter '85]

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- Maintain a (uniform) sample (w/o replacement) of size s
  from a stream of n items
  - Every subset of size s has equal probability to be the sample
- When the *i*-th item arrives
  - With probability s/i, use it to replace an item in the current sample chosen uniformly at ranfom
  - With probability 1 s/i, throw it away

## Reservoir Sampling from Distributed Streams

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Tracking i approximately? Sampling won't be uniform

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The coordinator could maintain a Bernoulli sample of size between s and O(s)

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(The lower sample is a sample with prob.  $2^{-i-1}$ )

- When the lower sample reaches size s, the coordinator broadcasts to advance to round  $i \leftarrow i + 1$ 
  - Discard the upper sample

Split the lower sample into a new lower sample and a higher sample



## Sampling from Distributed Streams: Analysis

- Communication cost of round *i*: O(k + s)
  - Expect to receive O(s) sampled items before round ends
  - Broadcast to end round: O(k)

[Cormode, Muthukrishnan, Yi, Zhang, PODS'10, JACM'12] [Woodruff, Tirthapura, DISC'11]

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- Number of rounds:  $O(\log(n/s))$ 
  - In round *i*, need  $\Theta(s)$  items being sampled to end round
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  - Each item has prob.  $2^{-i}$  to contribute: need  $\Theta(2^i s)$  items
- Communication:  $O((k + s) \log n)$ 
  - Can be improved to  $O(k \log_{k/s} n + s \log n)$
  - A matching lower bound

[Cormode, Muthukrishnan, Yi, Zhang, PODS'10, JACM'12] [Woodruff, Tirthapura, DISC'11]

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- $F_2$ :  $\tilde{O}(k^2/\varepsilon^2 + k^{1.5}/\varepsilon^4)$  [Cormode, Muthukrishnan, Yi, SODA'08]
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- $F_2$ :  $\tilde{\Omega}(k/\varepsilon^2)$  [Woodruff, Zhang, STOC'12]

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- $F_p, p > 1$ :  $\tilde{\Theta}(k^{p-1}/\text{poly}(\varepsilon))$  [Woodruff, Zhang, STOC'12]
- $F_0$  (distinct count):  $\tilde{\Theta}(k/\varepsilon^2)$  [Woodruff, Zhang, STOC'12]

- Entropy [Arackaparambil, Brody, Chakrabarti, ICALP'08]
- Heavy hitters and quantiles [Yi, Zhang, PODS'09]
  [Huang, Yi, Zhang, PODS'12]
- Sliding windows [Chan, Lam, Lee, Ting, STACS'10]
  [Cormode, Yi, SSDBM'12]

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- Does it have to be streaming?
  - If we don't care about space ...
  - Even if we care about space... streaming lower bounds do not apply!
- How to model deletions?
  - Competitive analysis? [Yi, Zhang, SODA'09]

# The Greater Picture: Distributed

- Motivated by database/networking applications
  - Adaptive filters [Olston, Jiang, Widom, SIGMOD'03]
  - A generic geometric approach [Scharfman et al. SIGMOD'06]
  - Prediction models [Cormode, Garofalakis, Muthukrishnan, Rastogi, SIGMOD'05]



network monitoring



sensor networks



cloud computing

environment monitoring

